Name: _____ Date: Jan 28 Hour: ___Alg 1___

Unit 5b Day 11: The Types of Numbers and Closed sets

Focus Question: What are the types of numbers that exist? Are they closed?

A. Sets

A **set** is any group of numbers. For example 4, 9, and 10 can make up a set. It is a set with 3 numbers. Some sets are very common and get names. For example 2, 4, 6, 8… is the set of even numbers.

Give the name of each set of numbers below.

1. 10, 20, 30, 40…. multiples of 10
2. 1, 4, 9, 16, 25…. perfect squares
3. 3, 5, 7, 11, 13, 17… primes

B. Closed

A set is considered "closed under *an operation*" if **any** two numbers from that original set are combined using the operation and the answer is still part of that original set.

If you can come up with a single time that the answer is not in the set, then it is not closed. The time that it doesn't work is called the counter-example.

For example, the original set of 4, 9, and 10 is NOT closed under addition because 4+9 = 13 and 13 is NOT in the original set. The set of even numbers is closed under addition because 4+6 = 10 and 10 is still an even number. Any two even numbers added together will **always** result in an even number.

Are the sets below **closed under addition**? If the answer is no, provide a counter-example.

1. 10, 20, 30, 40…. yes
2. 1, 4, 9, 16, 25…. No 1+4=5 5 is not a perfect sq.
3. 3, 5, 7, 11, 13, 17… No 3+5=8 8 is not prime

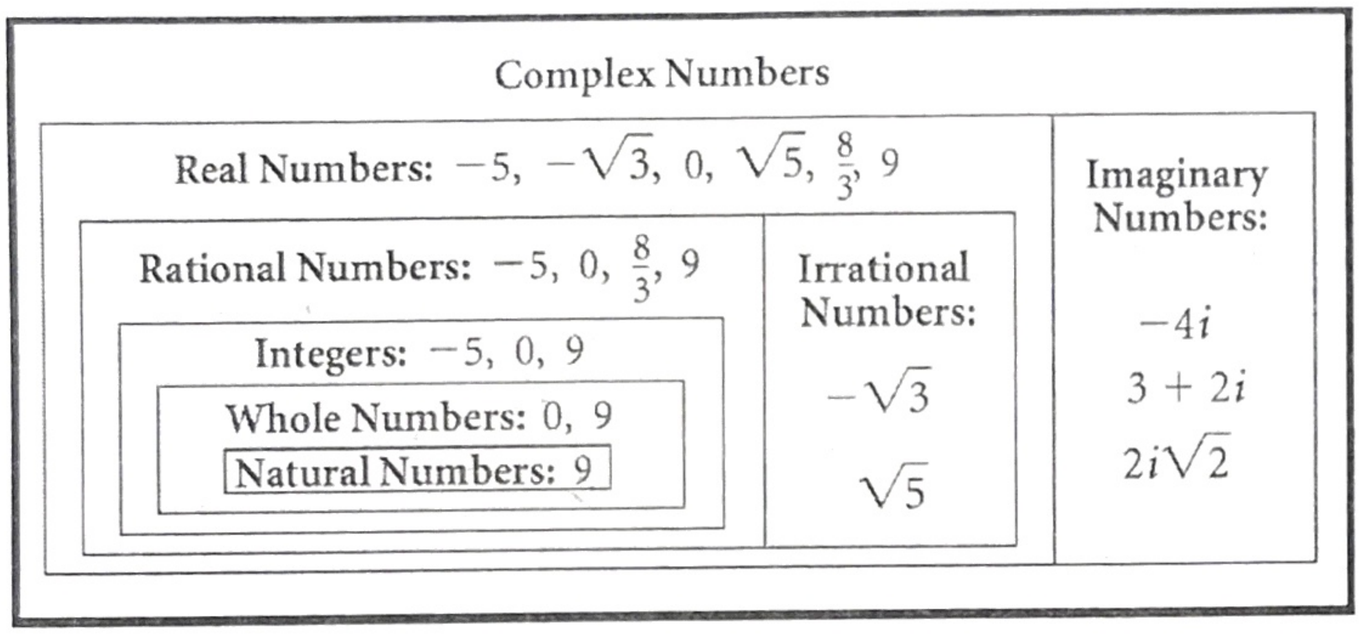
C. The Number System

All numbers are classified using The Complex Number System (all numbers are complex).
It is first decided if a number is Real or Imaginary. (We'll talk about these in a later unit!)
All real numbers are then classified as either rational or irrational.
We will be working only on the Rational Numbers today.

Set	Definition	Represented by …	Examples
Natural Numbers	The counting numbers	$\mathbb{N}$	1, 2, 5, 27, 30
(Whole Numbers)	The natural numbers with zero included	$\mathbb{N}_0$	1, 2, 5, 27, 30, 0
Integers	Positive or Negative Whole Numbers	$\mathbb{Z}$	1, 2, 5, 0, −2, −3,
Rationals	Any number that **CAN BE** written as a ratio of 2 integers (and the denominator is not zero) $\frac{}{0} = \emptyset$	$\mathbb{Q}$	$\frac{2}{1}, \frac{-3}{1}, \frac{3}{2}, \frac{-10}{7}$

Division Problem: Quotient

Below is a diagram of how the number system is organized. Classifying can be tricky because we are used to giving "the best" or most specific classification and forget that a number (or shape or "thing") can meet a more general definition as well.



1. Why is -5 listed as a rational and not just an integer?
   Integers are rational b/c you can put a 1 as the denominator
2. Why is 9 listed as an integer and not just a natural number?
   Integers can be positive or negative
3. True or false (if false give a counter example): All Natural numbers are rational.
   True all natural #'s can have a 1 put under them

4. True or false (if false give a counterexample): All Integers are natural.
   False -6 is an integer but it is not natural

D. The size of infinity

1. Are the natural numbers infinite? Explain.
   Yes, they go forever in 1 direction

2. Are the integers infinite? Explain.
   Yes, they go forever in 2 directions.

3. Are there different sizes of infinity? Explain.
   Yes the $\mathbb{N}$ infinity is smaller than the integers ∞.

4. Which has a bigger infinity, the integers or the rationals? Explain.
   b/c it divides the integers infinitely many times

E. Rational Numbers and Closed Sets

Question	Circle One	Example or Counter Example
Are the **natural** numbers closed under **addition**?	YES NO	$1+1=2$ $\mathbb{N}+\mathbb{N}=\mathbb{N}$
Are the **rational** numbers closed under **subtraction**?	YES NO	$\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$ $\mathbb{Q}-\mathbb{Q}=\mathbb{Q}$ $\frac{6}{10}-\frac{5}{10}$
Are the **integers** closed under **division**?	YES NO	$-6 \div -4 = \frac{3}{2}$ $\mathbb{Z} \div \mathbb{Z} =$ not $\mathbb{Z}$
Are the **natural** numbers closed under **multiplication**?	YES NO	$5 \cdot 2 = 10$ $\mathbb{N} \cdot \mathbb{N} = \mathbb{N}$
Are the **integers** closed under **subtraction**?	YES NO	$-4 - -6 = 2$ $\mathbb{Z} - \mathbb{Z} = \mathbb{N}$ ← but naturals are integers too
Are the **rational** numbers closed under **division**?	YES NO	$\frac{2}{3} \div \frac{6}{7} = \frac{7}{9}$ $\mathbb{Q} \div \mathbb{Q} = \mathbb{Q}$ $\frac{2}{3} \cdot \frac{7}{6} = \frac{14}{18}$