

Unit 5b Day 12 and 13: Operations with Polynomials

Focus Question: Are polynomials a closed set?

A. Review

1. What are polynomials? Give 3 examples (and classify them).

$5x^4 + 3x^2 - 4$ (4th degree trinomial)
 $7x^2 + 8x$ (2nd deg. binomial)
 $3x^4 + 2x$ (4th deg. binomial)

2. If you were asked to perform the following operation with polynomials, what would you look for?

$(5x^4 + 3x^2 - 4) + (7x^2 + 8x)$

Same exp & same variable ("like terms")

$5x^4 + 3x^2 - 4 + 7x^2 + 8x = 5x^4 + 10x^2 + 8x - 4$

3. Simplify the above expression and determine if the answer is still a polynomial.

Yes

4. Are polynomials closed under addition? If not, give a counter example.

Good Try $2x + 2x = 4x$
 (monomial) still polynomial

Yes

5. Simplify the expression $(6x^3 - 4x + 8) - (7x - 2)$.

$6x^3 - 4x + 8 - 7x + 2 = 6x^3 - 11x + 10$

6. Are polynomials closed under subtraction? If not, give a counter example.

Good Try $1x - 1x = 0$
 monomial constant still polynomials

Yes

$3x^2 \cdot 4x^3 = 12x^5$

B. Multiplying Polynomials

1. So far when you have multiplied polynomials, one of the factors has always been a monomial. For example, simplify $4(2x^3 - 7x)$. $4(2x^3) + 4(-7x) = 8x^3 - 28x$

2. What property did you use to simplify? Distribute

3. Multiply: $17 \cdot 32$. Explain your strategy.

4. Use the area model strategy at right to do $17 \cdot 32$

No matter which way you prefer, the idea is that both place values of the 32 get multiplied by both places values of the 17. The area model on the right can be much more helpful as you learn to multiply polynomials.

	10	+	7
30	$30 \cdot 10$		$30 \cdot 7$
+	300	+	210
2	$10 \cdot 2$		$7 \cdot 2$
	20	+	14

$300 + 210 + 20 + 14 = 544$

bin bin
5. $(2x^5 + 1)(3x^2 - 10)$

$$2x^5(3x^2 - 10) + 1(3x^2 - 10)$$

$$2x^5(3x^2) + 2x^5(-10) + 1(3x^2) + 1(-10)$$

$$6x^7 - 20x^5 + 3x^2 - 10$$

	$2x^5$	$+1$
$3x^2$	$2x^5 \cdot 3x^2$ $6x^7$	$3x^2 \cdot 1$ $3x^2$
-10	$2x^5 \cdot (-10)$ $-20x^5$	$-10 \cdot 1$ -10

$$6x^7 - 20x^5 + 3x^2 - 10$$

trin bin
6. $(5x^3 - 2x + 7)(4x^2 + 3)$

$$5x^3(4x^2 + 3) - 2x(4x^2 + 3) + 7(4x^2 + 3)$$

$$5x^3(4x^2) + 5x^3(3) - 2x(4x^2) - 2x(3) + 7(4x^2) + 7(3)$$

$$20x^5 + 15x^3 - 8x^3 - 6x + 28x^2 + 21$$

	$5x^3$	$-2x$	$+7$
$4x^2$	$5x^3 \cdot 4x^2$ $20x^5$	$4x^2 \cdot (-2x)$ $-8x^3$	$4x^2 \cdot 7$ $28x^2$
$+3$	$5x^3 \cdot 3$ $15x^3$	$3 \cdot (-2x)$ $-6x$	$3 \cdot 7$ 21

$$20x^5 + 7x^3 + 28x^2 - 6x + 21$$

7. $(4x^2 - 3)(6x^4 + 2x^2 + 1)$

$$4x^2(6x^4 + 2x^2 + 1) - 3(6x^4 + 2x^2 + 1)$$

$$4x^2(6x^4) + 4x^2(2x^2) + 4x^2(1) - 3(6x^4) - 3(2x^2) - 3(1)$$

$$24x^6 + 8x^4 + 4x^2 - 18x^4 - 6x^2 - 3$$

	$4x^2$	-3
$6x^4$	$24x^6$	$-18x^4$
$2x^2$	$8x^4$	$-6x^2$
$+1$	$4x^2$	-3

$$24x^6 - 10x^4 - 2x^2 - 3$$

$$24x^6 - 10x^4 - 2x^2 - 3$$

We'll now go back through those and show how they can all be done with the distributive property.

C. Use the Distributive Property to write each expression in expanded form.

1. $(3x^4 + 5)(x - 6)$

2. $(2d + 3)(3d + 1)$

$$3x^4(x - 6) + 5(x - 6)$$

$$3x^4(x) + 3x^4(-6) + 5(x) + 5(-6)$$

$$3x^5 - 18x^4 + 5x - 30$$

$$2d(3d + 1) + 3(3d + 1)$$

$$2d(3d) + 2d(1) + 3(3d) + 3(1)$$

$$6d^2 + 2d + 9d + 3$$

$$6d^2 + 11d + 3$$

3. $(4y^2 + y - 7)(2y - 1)$

4. $(7x - 4)(2x^3 + x - 5)$

$4y^2(2y-1) + y(2y-1) - 7(2y-1)$

$7x(2x^3+x-5) - 4(2x^3+x-5)$

$4y^2(2y) + 4y^2(-1) + y(2y) + y(-1) - 7(2y) - 7(-1)$

$7x(2x^3) + 7x(x) + 7x(-5) - 4(2x^3) - 4(x) - 4(-5)$

$8y^3 - 4y^2 + 2y^2 - y - 14y + 7$

$14x^4 + 7x^2 - 35x - 8x^3 - 4x + 20$

$8y^3 - 2y^2 - 15y + 7$

$14x^4 - 8x^3 + 7x^2 - 39x + 20$

5. Write and simplify a polynomial expression for the area of the triangle.

$A = \frac{bh}{2} = \frac{(x+1)(2x-8)}{2} = \frac{x(2x-8) + 1(2x-8)}{2}$

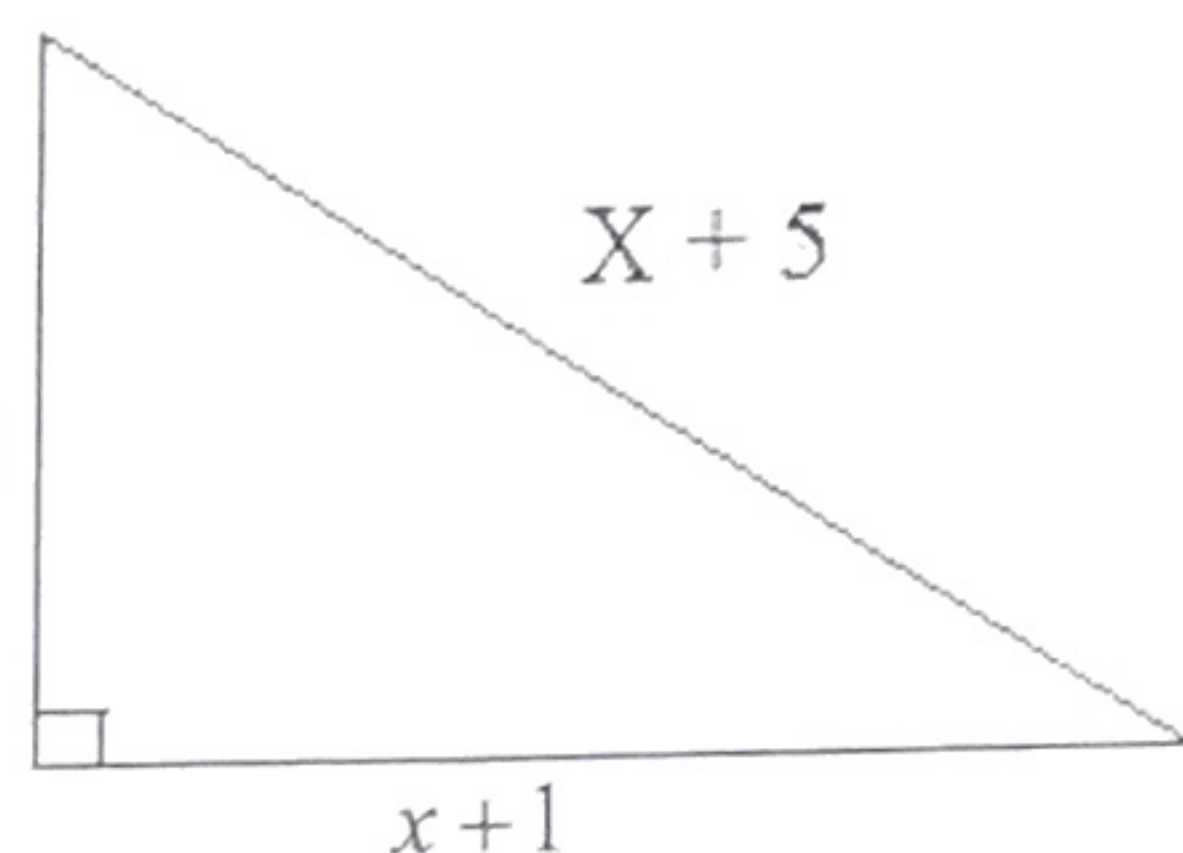
$2x^2 - 8x + 2x - 8$

$2x^2 - 6x - 8$

$\frac{2x^2 - 6x - 8}{2} = \frac{2(x^2 - 3x - 4)}{2}$

$x^2 - 3x - 4$

$x^2 - 3x - 4$



6. Are polynomials closed under multiplication? If not, give a counter-example.

Yes

D. Dividing polynomials

This is a skill that you learn much more about in algebra 2. But problem 5 in part C might make you

think that polynomials are closed under division because the final answer can be written as $\frac{2x^2 - 6x - 8}{2}$

(a very simple division of polynomials problem) or as $x^2 - 3x - 4$ which is still a polynomial. However,

when we start to work with quadratics we will see that sometimes you can divide $\frac{x^2 + 2x + 1}{x + 1}$ and

sometimes you cannot $\frac{x^2 + 2x - 1}{x + 1}$. Thus, polynomials are NOT closed under division. (It creates what

are called rational functions...more function families for you to learn in algebra 2!)

E. Closed Sets Comparison

1. For each set of numbers list the operations under which it is closed.

Set	Naturals	Integers	Rationals	Polynomials
Operations Under which The set is closed	Add multiply	Add Subt mult.	Add Subt. mult. divide	Add Subt Mult.

2. Polynomials function the most like which set of numbers? Explain why you believe this is so.

Integers, b/c the coefficients are integers

3. What do you notice is unique about rational numbers?

Closed under everything

4. Go back and look at the diagram of the number system on Day 11. How does your answer to number 3 help you understand the diagram.

