Name: ______________________ Date: Jan 31 ______________ Hour: __Alg 1____

## Unit 5b Day 14, 15, and 16: Decimals

*Focus Question: What are decimals? Are they their own set? How do I turn them back into their actual form?*

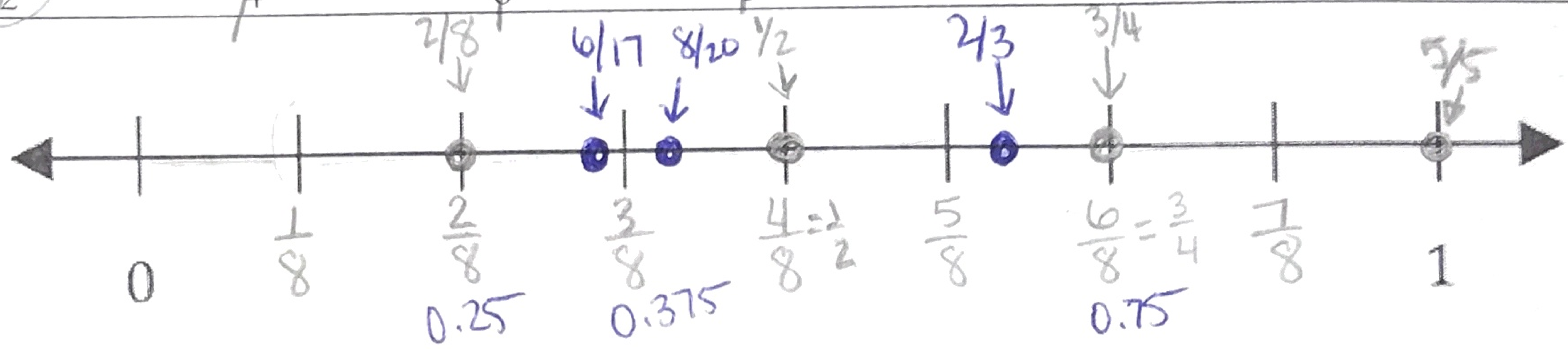
A. Review:

1. What was the definition of rational number?

A # that CAN be written as a fraction of integers (+ or − whole #'s)

2. Place each rational number in its correct place on the number line below.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{2}{8}$ | $\frac{5}{5}$ | $\frac{2}{3}$ | $\frac{6}{17}$ | $\frac{8}{20}$ |




3. What made some of the rational numbers difficult to place?

$\frac{2}{3}$ $\frac{6}{17}$ $\frac{8}{20}$ the denominator is not a multiple or factor of 8

4. What was a strategy you used to help place them correctly?

$2 \div 3 = 0.\overline{6}$ $5 \div 8 = 0.625$ ÷ to get decimal place value

$6 \div 17 \approx 0.35$ $8 \div 20 = 0.4$

**B. What are decimals?**

Decimals come from long division of integers therefore, **decimals are** rational **numbers** (THEY ARE <u>**NOT**</u> THEIR OWN SET OF NUMBERS IN THE COMPLEX NUMBER SYSTEM). Decimals are simply a **more convenient way** of expressing a rational number. (They are easier to place on a number line). Decimals come in two types.

1. What do all of the following decimals have in common? They stop $\boxed{\text{Terminating}}$

0.8 -0.75 9.61 -4.3 -2.659 12.012

2. What do all of the following decimals have in common? They repeat $\boxed{\text{Repeating}}$

6.555… -2.121212… 0.784784784… -0.2828282…

$6.\overline{5}$ $-2.\overline{12}$ $0.\overline{784}$ $-0.\overline{28}$

Write each decimal above correctly.

Natural numbers and Integers are Rational numbers. Because decimals are also rational numbers, natural numbers and integers must also be able to be written as decimals.

3. Write the following numbers as decimals.

| 16 | -5 | 2 | -275 |
|---|---|---|---|
| 16.0000…. | -5.0 | 2.0 | -275.0 |

4. Which type of decimal (repeating or terminating) are natural numbers and integers?

Trick Question! There's debate!

T

5. Correctly write each rational number as a decimal. Then tell which type of decimal (terminating or repeating). R

R $\frac{8}{11} = 0.\overline{72}$

T $\frac{1}{4} = 0.25$

R $\frac{5}{7} = 0.\overline{714285}$

T $\frac{8}{10} = 0.8$

T $\frac{248}{50} = 4.96$

R $\frac{56}{12} = 4.\overline{6}$

C. **Terminating Decimals Back to Fractions: Use Place Value**

1. Fill in the blank for the place value

*Decimal Place values* 0.1234567

ones, tenths, hundredths, thousandths, ten thousandths, hundred thousandths, Millionths, ten millionths

$0.5 = \frac{5}{10}$

2. How do you use place value to turn a terminating decimal back into a fraction?

the place value is the denominator

3. Examples: Turn each terminating decimal into a rational number.

a. 0.127 $\frac{127}{1000}$

b. -0.3 $\frac{-3}{10}$

c. 0.81 $\frac{81}{100}$

d. -0.6317 $\frac{-6317}{10000}$

Those were easy because all of them had no whole numbers. While it is nice to know mixed numbers to put numbers on a number line, upper level math teacher prefer answers that are improper fractions.

4. What is an improper fraction? numerator is bigger than the denominator

5. How do you turn a mixed number into an improper fraction?

$6\frac{1}{2} \rightarrow 6 + \frac{1}{2} \rightarrow \frac{12}{2} + \frac{1}{2} \rightarrow \boxed{\frac{13}{2}}$

6. Turn the following mixed numbers into fractions.

a. $4\frac{1}{2} = \boxed{\frac{9}{2}}$ $\quad \frac{8}{2} + \frac{1}{2}$

b. $-6\frac{3}{4} = \boxed{\frac{-27}{4}}$ $\quad -\frac{24}{4} - \frac{3}{4}$

c. $9\frac{5}{8} = \boxed{\frac{77}{8}}$ $\quad \frac{72}{8} + \frac{5}{8}$

d. $-10\frac{2}{3} = \boxed{\frac{-32}{3}}$ $\quad -\frac{30}{3} - \frac{2}{3}$

7. Turn the following terminating decimals into fractions.

a. 8.7 $\quad 8 + \frac{7}{10} \quad \frac{80}{10} + \frac{7}{10} \quad \boxed{\frac{87}{10}}$

b. -2.13 $\quad -\left(2 + \frac{13}{100}\right) \quad -\left(\frac{200}{100} + \frac{13}{100}\right) \quad \boxed{\frac{-213}{100}}$

c. 5.173 $\quad 5 + \frac{173}{1000} \quad \frac{5000}{1000} + \frac{173}{1000} \quad \boxed{\frac{5173}{1000}}$

d. -1.41 $\quad -\left(1 + \frac{41}{100}\right) \quad -\left(\frac{100}{100} + \frac{41}{100}\right) \quad \boxed{\frac{-141}{100}}$

e. -4.3 $\quad -\left(4 + \frac{3}{10}\right) \quad -\left(\frac{40}{10} + \frac{3}{10}\right) \quad \boxed{\frac{-43}{10}}$

Those still weren't too hard because they didn't require reducing.

8. Show work to reduce the following fractions (We will show work in a way that will help you simplify rational expressions in the future.)

$\frac{4}{100} = \frac{\cancel{4}\cdot 1}{\cancel{4}\cdot 25} = \boxed{\frac{1}{25}}$

$\frac{9}{147} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 49} = \boxed{\frac{3}{49}}$

$\frac{42}{24} = \frac{\cancel{6}\cdot 7}{\cancel{6}\cdot 4} = \boxed{\frac{7}{4}}$

$\frac{95}{30} = \frac{\cancel{5}\cdot 19}{\cancel{5}\cdot 6} = \boxed{\frac{19}{6}}$

**Quick Divisility Rules**

| A # is divisible by... | if... | Example |
|---|---|---|
| 2 | Its even<br>~~It's even~~ | 4, 8, 106, 2012 |
| 3 | the sum of its numbers is divisible by 3 | 243 is divisible by 3 because 2+4+3 = 9 and 9 is divisible by 3<br>761 is NOT divisible by 3 because 7+6+1 = 14 and 14 is NOT divisible by 3 |
| 4 | the last 2 digits are divisible by 4 | 236 is divisible by 4 because 36 is divisble by 4<br>522 is NOT divisble by 4 because 22 is NOT divisble by 4 |
| 5 | ends in 0 or 5 | 10, 2020 |
| 6 | ~~It ends in 0 or 5~~<br>It is divisible by both 2 and 3 | |
| 9 | Sum is divisible by 9 | |
| 10 | ends in 10 | |

**Putting it all together**

9. Turn each terminating decimal into a fraction. Remember to reduce your fraction.

a. 8.4 $= \boxed{\frac{42}{5}}$

$8 + \frac{4}{10}$

$8 + \frac{2}{5}$

$\frac{40}{5} + \frac{2}{5}$

b. 4. 85 $= \boxed{\frac{97}{20}}$

$4 + \frac{85}{100}$

$4 + \frac{17}{20}$

$\frac{80}{20} + \frac{17}{20}$

c. -0.963

$\boxed{\frac{-963}{1000}}$

d. – 5.24 $= \boxed{\frac{-131}{25}}$

$-\left(5 + \frac{24}{100}\right)$

$-\left(5 + \frac{6}{25}\right)$

$-\left(\frac{125}{25} + \frac{6}{25}\right)$

**D. Repeating Decimals back to fractions**

1. What does repeating mean? A# goes on forever

2. Which of the following numbers is a repeating decimal? Explain.

a. $\boxed{5.676767\ldots}$ b. 2.3131131113…

$\boxed{5.\overline{67}}$

The 67 repeats forever

3. Are repeating decimals rational numbers? Explain.

Yes

4. Circle all of the terms below that are repeating decimals.

~~-6.13~~ $\quad \frac{-3}{13} \quad$ ~~0.5~~ $\quad 6.\overline{25} \quad 2 \quad \frac{8}{3}$

2.00....

$2.\overline{0}$

**The Mathematician's Way**

Because repeating decimals came from rational numbers, there must be a way to turn them back into their rational equivalent. The following is the mathematically correct way to turn any repeating decimal into its fractional equivalent. It works for ALL repeating decimals.

To turn $0.\overline{12}$ into its equivalent fraction....

Let $x = 0.121212\ldots$

5. What is $100x$? 12.121212....

6. Complete the subtraction.

7. Complete the process by solving for $x$.

$$\begin{array}{r} 100x = 12.121212\ldots \\ -\quad x = \ \ 0.121212\ldots \\ \hline 99x = 12 \end{array}$$

$$\frac{99x}{99} = \frac{12}{99}$$

$$x = \frac{12}{99}$$

$$x = \frac{4}{33}$$

8. Remember x = 0.121212...

and now x = 4/33

therefore $0.\overline{12} = \boxed{\frac{4}{33}}$

9. Why do you think this method starts out by multiplying by 100?

it repeats in the hundredths place

10. Turn the following into their fractional equivalent.

a. $5.\overline{7}$

b. $2.1\overline{3}$

$$\begin{array}{r} 10x = 57.7777\ldots \\ -\quad x = 5.7777\ldots \\ \hline 9x = 52 \end{array}$$

$$\frac{9x}{9} = \frac{52}{9}$$

$$\boxed{x = \frac{52}{9}}$$

$x = 2.13333\ldots$

$$\begin{array}{r} 100x = 213.3333\ldots \\ -\quad 10x = 21.3333\ldots \\ \hline 90x = 192 \end{array}$$

$$\frac{90x}{90} = \frac{192}{90}$$

$$x = \frac{192}{90}$$

$$\boxed{x = \frac{32}{15}}$$

**The shortcut way**

There is short cut that works for **MOST** repeating decimals. Find it by looking for a pattern when filling in the table below using a calculator.

| Fraction | Decimal | Fraction | Decimal |
|---|---|---|---|
| $\frac{1}{9}$ | $0.\overline{1}$ | $\frac{4}{99}$ | $0.\overline{04}$ |
| $\frac{16}{99}$ | $0.\overline{16}$ | $\frac{4}{9}$ | $0.\overline{4}$ |
| $\frac{512}{999}$ | $0.\overline{512}$ | $\frac{5}{9}$ | $0.\overline{5}$ |
| $\frac{8}{9}$ | $0.\overline{8}$ | $\frac{7813}{9999}$ | $0.\overline{7813}$ |

*11. Describe any patterns you see in your table.

*The repeating part becomes the numerator

*The denominator will be the place value minus 1

12. Turn the following repeating decimals into fractions.

$0.\overline{3}$ $\quad -3.\overline{09}$ $\quad -1.\overline{42}$ $\quad 2.\overline{73}$

$\frac{3}{10-1}$ or $\frac{3}{9}$ $\quad \boxed{\frac{1}{3}}$

$-\left(3+\frac{9}{100-1}\right)$ $\quad -\left(3+\frac{9}{99}\right)$ or $-\left(3+\frac{1}{11}\right)$ $\quad -\left(\frac{33}{11}+\frac{1}{11}\right)$ $\quad \boxed{-\frac{34}{11}}$

$-\left(1+\frac{42}{100-1}\right)$ $\quad -\left(1+\frac{42}{99}\right)$ or $-\left(1+\frac{14}{33}\right)$ $\quad -\left(\frac{33}{33}+\frac{14}{33}\right)$ $\quad \boxed{-\frac{47}{33}}$

$2+\frac{73}{100-1}$ $\quad 2+\frac{73}{99}$ $\quad \frac{198}{99}+\frac{73}{99}$ $\quad \boxed{\frac{271}{99}}$

13. The shortcut does NOT work for all repeating decimals. It would not work for $2.1\overline{3}$ from 10b above. It would also not be true to say $0.62\overline{3} = \frac{623}{999}$ . Explain what makes these repeating decimal different (and therefore why the shortcut doesn't work).

The whole decimal portion does not repeat

14. Interesting Mathematical concept…What does $0.\overline{9}$ equal? 1 Prove it! $0.9999\ldots$

Proof 1

short cut

$0.\overline{9} = \frac{9}{10-1}$

$= \frac{9}{9}$

$= 1$

Proof 2

correct way

$$\begin{aligned} 10x &= 9.9999\ldots \\ - \quad x &= 0.9999\ldots \\ \hline 9x &= 9 \\ \frac{9x}{9} &= \frac{9}{9} \\ x &= 1 \end{aligned}$$

Proof 3

$3 \cdot \frac{1}{3} = 0.\overline{3} \cdot 3$

$1 = 0.\overline{9}$

$1 =$

$1 = x^0$

$1 = 0.\overline{9}$