Name: ______________________ Date: Feb 6 ______________ Hour: ____Alg 1__

Unit 5 Day 18 and 19: Operations with Irrational Numbers

Focus Question: Are irrational numbers closed under addition, subtraction, multiplication, or division?

A. Simplifying Radicals Review

Simplify each of the radicals below (remember radicals can be written with an exponent and therefore follow rules of exponents!).

1. $\sqrt{20}$
   $\sqrt{4\cdot5}$
   $2\sqrt{5}$
2. $\sqrt{288}$
   $\sqrt{144\cdot2}$
   $12\sqrt{2}$
3. $\sqrt{375}$
   $\sqrt{25\cdot15}$
   $5\sqrt{15}$
4. $\sqrt{99}$
   $\sqrt{9\cdot11}$
   $3\sqrt{11}$

5. Is a simplified radical rational or irrational? Explain.

Irrational, decimal goes on forever w/no pattern

6. Each simplified radical has two factors: an integer factor and an irrational factor. Which one do we put first? What operation is implied between the two factors?

Rational factor 1st, multiplication

B. Multiplying Radicals…You've done this before , so it's still review =)

1. Is $\sqrt{9}\cdot\sqrt{4}=\sqrt{36}$?

$3\cdot2=6$
$6=6$ True

$\sqrt{9}\cdot\sqrt{4}$
$9^{1/2}\cdot4^{1/2}=(9\cdot4)^{1/2}$ or $36^{1/2}$

When multiplying radicals you can write one longer radical and move all of the multiplication under the radical. You must remember to simplify your answer. You are allowed to do this because the bases (number under the radical) really have the same exponent.

2. Simplify the following expressions.

b. $\sqrt{5}\cdot\sqrt{10}=\boxed{5\sqrt{2}}$
$\sqrt{5\cdot10}$
$\sqrt{50}$
$\sqrt{25\cdot2}$

b. $\sqrt{14}\cdot\sqrt{2}=\boxed{2\sqrt{7}}$
$\sqrt{14\cdot2}$
$\sqrt{28}$
$\sqrt{4\cdot7}$

c. $\sqrt{3}\cdot\sqrt{27}=\boxed{\pm9}$
$\sqrt{3\cdot27}$
$\sqrt{81}$

d. $\sqrt{15}\cdot\sqrt{2}$
$\sqrt{15\cdot2}$
$\boxed{\sqrt{30}}$

3. Which example(s) above shows **that irrationals are NOT closed under multiplication**?

c   9 is not irrational

4. When two simplified radicals are being multiplied together, what operation is indicated between all factors?

multiplication

5. Are you allowed to perform this operation in any order? Yes b/c of the commutative property

6. Rewrite and simplify each expression.

a. $-4\sqrt{7}\cdot2\sqrt{3}$
$-4\cdot2\sqrt{7\cdot3}$
$\boxed{-8\sqrt{21}}$

b. $10\sqrt{18}\cdot6\sqrt{3}=\boxed{180\sqrt{6}}$
$10\cdot6\sqrt{18\cdot3}$
$60\sqrt{54}$
$60\sqrt{9\cdot6}$
$60\cdot3\sqrt{6}$

c. $2\sqrt{20}\cdot4\sqrt{5}$
$2\cdot4\cdot\sqrt{20\cdot5}$
$8\cdot\sqrt{100}$
$8\cdot10$
$\boxed{\pm80}$

C. **Dividing Radicals…still review from first part of unit!**

Multiplication and division are simply inverse operations so it is also true that you can make one large radical and move all division under the radical. (This is actually allowed because the bases have the same exponent again.)

1. Simply each expression

a. $\frac{\sqrt{18}}{\sqrt{2}}$ $\sqrt{\frac{18}{2}}$ $\sqrt{9} = \boxed{\pm 3}$

b. $\frac{\sqrt{24}}{\sqrt{3}} = \boxed{2\sqrt{2}}$ $\sqrt{\frac{24}{3}} = \sqrt{8}$ $\sqrt{4 \cdot 2}$

c. $\frac{\sqrt{90}}{\sqrt{6}} = \boxed{\sqrt{15}}$

2. Which expression(s) above shows **that irrationals are NOT closed under division**?

a, ± 3 is not irrational

3. Rewrite each division problem and simplify

a. $\frac{27\sqrt{6}}{3\sqrt{2}}$ $\frac{27}{3} \cdot \sqrt{\frac{6}{2}}$ $\boxed{9\sqrt{3}}$

b. $\frac{40\sqrt{21}}{10\sqrt{3}}$ $\frac{40}{10}\sqrt{\frac{21}{3}}$ $\boxed{4\sqrt{7}}$

c. $\frac{5\sqrt{26}}{10\sqrt{13}}$ $\frac{1}{2}\sqrt{2}$ $\frac{1}{2} \cdot \frac{\sqrt{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$

Rationalizing Denominators (THIS IS NEW!).

Sometimes the numerator and denominator do not divide into an integer as all of the problems we have done so far. In this case you need to "rationalize the denominator."

4. What do you think this term means?

make the denominator rational

5. The easiest way to turn an irrational number into a rational number is to multiply it by itself. Remember that if you multiply in the denominator, you must also multiply in the numerator so that you are not changing the number. Rationalize each denominator below.

a. $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

b. $\frac{6}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{6\sqrt{7}}{7}}$

c. $\frac{2}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{5 \cdot 3}$ $\boxed{\frac{2\sqrt{3}}{15}}$

6. Simplify each expression below.

a. $\frac{2\sqrt{6}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{2\sqrt{60}}{10}$ $\sqrt{60}$ $\sqrt{4 \cdot 15}$ $2\sqrt{15}$ $\frac{2 \cdot 2\sqrt{15}}{10} = \frac{4\sqrt{15}}{10}$ $\boxed{\frac{2\sqrt{15}}{5}}$

b. $\frac{6\sqrt{3}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{15}}{2 \cdot 5}$ $\frac{6\sqrt{15}}{10} = \boxed{\frac{3\sqrt{15}}{5}}$

c. $\frac{7\sqrt{12}}{21\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}} = \frac{7\sqrt{216}}{21 \cdot 18}$ $\frac{7\sqrt{216}}{378} = \frac{7\sqrt{36 \cdot 6}}{378}$ $\frac{7 \cdot 6 \cdot \sqrt{6}}{378} = \frac{42\sqrt{6}}{378}$ or $\boxed{\frac{\sqrt{6}}{9}}$

D. Addition and Subtraction of Irrationals…We've done this before =)

1. "Like terms" Review: Simplify each expression

a. $6x + 3x^2 - 4x$ b. $7y + 4y - 2x$ c. $7x^2 + 4x - 2x + 3x^2$

$3x^2 + 2x$ $11y - 2x$ $10x^2 + 2x$

2. What makes "like terms?"

Same base & same exponent

"Like" Square Roots

Like Square Roots:	$\sqrt{3},\ 4\sqrt{3},\ -2\sqrt{3}$
NOT Like Square Roots:	$\sqrt{3},\ 4\sqrt{5},\ -2\sqrt{7}$

3. Based on the examples above, what do you think makes square roots "like."?

Same base, same exponent (½)

4. Just like terms, square roots can only be combined if they are "like." You treat the integer factor like a coefficient and the irrational factor like the variable. If possible, simplify the expression.

a. $3\sqrt{6} + 2\sqrt{6} = 5\sqrt{6}$

b. $-7\sqrt{2} + 3\sqrt{10}$

Can't simplify the 2 & 10 are not same base

c. $-3\sqrt{7} - 5\sqrt{7} = -8\sqrt{7}$

d. $4\sqrt{10} - 4\sqrt{10} = 0$

e. $-2\sqrt{5} + 2\sqrt{5} = 0$

f. $-\sqrt{15} + 6\sqrt{15} = 5\sqrt{15}$

5. Which problem above shows that **irrationals are NOT closed under addition**?

e, 0 is not irrational

6. Which problem above shows that **irrationals are NOT closed under subtraction**?

d 0 is not irrational

7. Under what operations are irrationals closed?

None

B. **Harder Addition and subtraction of irrationals**
Some irrational numbers may not look like they are "like" so you must remember to simplify them first! Simplify each expression.

$\sqrt{50}+\sqrt{18}$

$\sqrt{25\cdot2}+\sqrt{9\cdot2}$

$5\sqrt{2}+3\sqrt{2}$

$\boxed{8\sqrt{2}}$

$3\sqrt{28}-\sqrt{63}$ $=\boxed{3\sqrt{7}}$

$3\cdot\sqrt{4\cdot7}-\sqrt{9\cdot7}$

$3\cdot2\sqrt{7}-3\sqrt{7}$

$6\sqrt{7}-3\sqrt{7}$

$\sqrt{8}+\sqrt{18}$

$\sqrt{4\cdot2}+\sqrt{9\cdot2}$

$2\sqrt{2}+3\sqrt{2}$

$\boxed{5\sqrt{2}}$

$\sqrt{75}-\sqrt{20}$

$\sqrt{25\cdot3}-\sqrt{4\cdot5}$

$\boxed{5\sqrt{3}-2\sqrt{5}}$

$\sqrt{27}+\sqrt{48}-2\sqrt{3}$

$\sqrt{9\cdot3}+\sqrt{16\cdot3}-2\sqrt{3}$

$3\sqrt{3}+4\sqrt{3}-2\sqrt{3}$

$\boxed{5\sqrt{3}}$

$\ast\ -5\sqrt{44}+2\sqrt{99}$ $=\boxed{-4\sqrt{11}}$

$-5\sqrt{4\cdot11}+2\sqrt{9\cdot11}$

$-5\cdot2\sqrt{11}+2\cdot3\sqrt{11}$

$-10\sqrt{11}+6\sqrt{11}$

$3\sqrt{72}+2\sqrt{75}-3\sqrt{27}+\sqrt{108}$

$3\sqrt{36\cdot2}+2\sqrt{25\cdot3}-3\sqrt{9\cdot3}+\sqrt{36\cdot3}$

$3\cdot6\sqrt{2}+2\cdot5\sqrt{3}-3\cdot3\sqrt{3}+6\sqrt{3}$

$18\sqrt{2}+\underbrace{10\sqrt{3}-9\sqrt{3}+6\sqrt{3}}$ $=\boxed{7\sqrt{3}+18\sqrt{2}}$

$\sqrt{250}-\sqrt{135}-\sqrt{40}+\sqrt{735}$

$\sqrt{25\cdot10}-\sqrt{9\cdot15}-\sqrt{4\cdot10}+\sqrt{49\cdot15}$

$5\sqrt{10}-3\sqrt{15}-2\sqrt{10}+7\sqrt{15}$

$\boxed{3\sqrt{10}+4\sqrt{15}}$