Name: ______________________ Date: Jan 9 ______________ Hour: ___Alg 1___

Unit 5a Day 3, 4, and 5: Working with Square Roots

Focus Questions: What is a good estimate when the side length is not a perfect square?
Why is square root the inverse of squared?
How do I simplify radicals?

$\sqrt{25}$	$\sqrt{144}$	$\sqrt{225}$	$\sqrt{81}$	$\sqrt{1}$
$\sqrt{9}$	$\sqrt{196}$	$\sqrt{4}$	$\sqrt{64}$	$\sqrt{100}$
$\sqrt{121}$	$\sqrt{49}$	$\sqrt{169}$	$\sqrt{16}$	$\sqrt{36}$

A. Perfect Squares
Place each of the square roots in the box on the number line below.

$\sqrt{1}$ $\sqrt{4}$ $\sqrt{9}$ $\sqrt{16}$ $\sqrt{25}$ $\sqrt{36}$ $\sqrt{49}$ $\sqrt{64}$ $\sqrt{81}$ $\sqrt{100}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{169}$ $\sqrt{196}$ $\sqrt{225}$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

B. "Non- Perfect" Squares
Not all squares are "perfect." Previously we found the area of these squares. Fill in the table for their area and side length. Then estimate their side length.

Square	Area	Exact Side length	Section of the number line	Estimate of the side length
$\sqrt{2}$	2	$S^2=A$ $\sqrt{S^2}=\sqrt{2}$ $S=\sqrt{2}$	$\sqrt{1}$ +1 $\sqrt{2}$ +2 $\sqrt{4}$ 1 $\frac{1}{3}$ 2	≈ 1.33
$\sqrt{5}$	5	$S^2=A$ $\sqrt{S^2}=\sqrt{5}$ $S=\sqrt{5}$	$\sqrt{4}$ +1 $\sqrt{5}$ +4 $\sqrt{9}$ 2 $\frac{1}{5}$ 3	≈ 2.2
B $\sqrt{8}$	8	$S^2=A$ $\sqrt{S^2}=\sqrt{8}$ $S=\sqrt{8}$	$\sqrt{4}$ +4 $\sqrt{8}$ +1 $\sqrt{9}$ 2 $\frac{4}{5}$ 3	≈ 2.8
E $\sqrt{10}$	10	$S=\sqrt{10}$	$\sqrt{9}$ +1 $\sqrt{10}$ +6 $\sqrt{16}$ 3 $\frac{1}{7}$ 4	≈ 3.15

These numbers look "weird" as answers because they are a new type of number for you. When there is a square root of a non-perfect square, for example $\sqrt{2}$, $\sqrt{6}$, or $\sqrt{94}$ they are called irrational. We will learn about classifying numbers later in the unit. For now we will learn to work with them and write them in different ways.

ut the following on the number line and give an estimate for each $\sqrt{50}$ $\sqrt{90} \approx 10.68$ $\sqrt{114}$

$\sqrt{49}$ $\sqrt{50} \approx 7.07$ $\sqrt{64}$ $\sqrt{81}$ +9 $\sqrt{90} \approx 9.45$ +10 $\sqrt{100}$ +14 $\sqrt{114}$ +7 $\sqrt{121}$

7 8 9 10 11 12

C. The Inverse of Squared

1. Solve: $x + 8 = -5$
   $-8 \quad -8$
   $x + 0 =$
   $\boxed{x = -13}$

   The inverse of add is subtract and the inverse of subtract is add. These are inverse operations because they create the additive identity (which is 0). This is the number you can add to/subtract from any number and not change its value.

2. Solve: $4x = -24$
   $\frac{4x}{4} = \frac{-24}{4}$
   $1x$
   $\boxed{x = -6}$

   The inverse of multiply is divide and the inverse of divide is multiply. These are inverse operations because they create the multiplicative identity (which is 1). This is the number you can multiply/divide any number by and not change its value.

3. Solve $2x^2 - 6 = 12$
   $+6 \quad +6$
   $\frac{2x^2}{2} = \frac{18}{2}$
   $\sqrt{x^2} = \sqrt{9}$
   $\boxed{x = \pm 3}$
   $(x^2)^? =$

4. When you solved #1, 2, and 3, what is the final exponent on x? 1

5. In #1 and 2, x started as degree 1. But in #3, x started as degree 2. So you took a power of 2 (squared) and did something to it in order to make it a power of 1.
   What are some rules we know about changing powers…

product rule: example $x^2 \cdot x^3 = x^{2+3} = x^5$

quotient rule: Example $m^8 \div m^2 = m^{8-2} = m^6$

power rule: Example $(h^5)^4 = h^{5 \cdot 4} = h^{20}$

The **identity exponent** is 1 because you can raise any base to it and it doesn't change the value.

6. Use the power rule to solve for x. $(a^2)^x = a^1$
   $a^{2x} = a^1$
   $\frac{2x}{2} = \frac{1}{2}$
   $x = \frac{1}{2}$

Taking the square root creates the identity exponent and raising squared to the 1/2 power creates the identity exponent. This means that square root can also be written as the 1/2 power and squared and square root are inverse operations because they create the identity exponent.

7. Re-write and simplify each of the following if possible.

a. $81^{\frac{1}{2}}$ $\sqrt{81}$ $\boxed{\pm 9}$

b. $225^{\frac{1}{2}}$ $\sqrt{225}$ $\boxed{\pm 15}$

c. $625^{\frac{1}{2}}$ $\sqrt{625}$ $\boxed{\pm 25}$

d. $200^{\frac{1}{2}}$ $\boxed{\sqrt{200}}$

e. $40^{\frac{1}{2}}$ $\boxed{\sqrt{40}}$

X·2

D. Operations with radicals

When the base (or number under a radical) is a perfect square, it is easiest to think of the problem as "square root of," but when you are working with non-perfect squares, it is typically easier to think of it as "the ½ power" and use the rules of exponents. (Especially the rule for different bases with the same exponent: $4^3 \cdot 6^3 =$ $(4\cdot6)^3$) $=24^3$

2x ~~+~~ multiply is implied

1. One of the following simplifies, which one and why?

a. $\sqrt{20}\cdot\sqrt{5}$ = 10 they have the same exponent

$20^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$

$(20\cdot5)^{1/2}$

$100^{1/2}$ or $\sqrt{100}$

b. $7\cdot\sqrt{3}$

$7^1 \cdot 3^{\frac{1}{2}}$

" $7\sqrt{3}$ (irrational # that has a rational factor)

" $\sqrt{3}\cdot 7$ same thing

2. It seemed like we could not simplify $200^{\frac{1}{2}}$ and $40^{\frac{1}{2}}$ but if we use our rules of exponents, we can see the following:

$200^{\frac{1}{2}}$ or $\sqrt{200}$

$(100\cdot2)^{\frac{1}{2}}$ or $\sqrt{100\cdot2}$

$100^{\frac{1}{2}}\cdot2^{\frac{1}{2}}$ or $\sqrt{100}\cdot\sqrt{2}$

$10\sqrt{2}$

Simplify $40^{\frac{1}{2}}$

$\sqrt{40}$

$\sqrt{4\cdot10}$

$\sqrt{4}\cdot\sqrt{10}$

$\boxed{2\sqrt{10}}$

Simplify $\sqrt{72}$

$\sqrt{36\cdot2}$

$\sqrt{36}\cdot\sqrt{2}$

$\boxed{6\sqrt{2}}$

$\sqrt{72}$

$\sqrt{9\cdot8}$

$\sqrt{9}\cdot\sqrt{8}$

$3\sqrt{8}$

$3\sqrt{4\cdot2}$

$3\sqrt{4}\cdot\sqrt{2}$

$3\cdot2\cdot\sqrt{2}$

A square root can be simplified if it has a perfect square as a factor.

3. Which of the following lengths can be simplified: $\sqrt{30}$, $\sqrt{10}$, $\sqrt{18}$? Explain.

perfect sq. 1, 4, 9, 16, 25, 36, 49 . . .

$\sqrt{9\cdot2}$

it has a factor that's a perfect sq.

4. Simplify each of the following (remember to think about the exponent on the base to know what you can and can't multiply and divide). (same exponent)

a. $\sqrt{5}\cdot\sqrt{10} = \boxed{5\sqrt{2}}$

$\sqrt{5\cdot10}$

$\sqrt{50}$

$\sqrt{25\cdot2}$

$\sqrt{25}\cdot\sqrt{2}$

b. $\sqrt{14}\cdot\sqrt{2} = \boxed{2\sqrt{7}}$

$\sqrt{14\cdot2}$

$\sqrt{28}$

$\sqrt{4\cdot7}$

$\sqrt{4}\cdot\sqrt{7}$

c. $\sqrt{3}\cdot\sqrt{27} = \boxed{\pm9}$

$\sqrt{3\cdot27}$

$\sqrt{81}$

d. $\sqrt{15}\cdot\sqrt{2} = \boxed{\sqrt{30}}$

$\sqrt{15\cdot2}$

$\sqrt{30}$

e. $-4\sqrt{7}\cdot2\sqrt{3}$

$-4\cdot2\cdot\sqrt{7}\cdot\sqrt{3}$

$-8\sqrt{7\cdot3}$

$\boxed{-8\sqrt{21}}$

f. $10\sqrt{18}\cdot6\sqrt{3}$

$10\cdot6\sqrt{18\cdot3}$

$60\sqrt{54}$

$60\sqrt{9\cdot6}$

$60\cdot\sqrt{9}\cdot\sqrt{6}$

$60\cdot3\cdot\sqrt{6}$

$\boxed{180\sqrt{6}}$

g. $2\sqrt{20}\cdot4\sqrt{5}$

$2\cdot4\cdot\sqrt{20\cdot5}$

$8\sqrt{100}$

$8\cdot10$

$\boxed{\pm80}$

h. $(3\sqrt{2})^2$ $\boxed{\pm 18}$

$3^2 \sqrt{2}^2$

$9 \cdot 2$

$(3\sqrt{2})(3\sqrt{2})$

$3 \cdot 3 \sqrt{2 \cdot 2}$

$9\sqrt{4}$

$9 \cdot 2$

i. $(-5\sqrt{3})^2$

$(-5)^2 \sqrt{3}^2$

$25 \cdot 3$

$\boxed{\pm 75}$

j. $\dfrac{\sqrt{90}}{\sqrt{6}}$

$\sqrt{\dfrac{90}{6}}$

$\boxed{\sqrt{15}}$

k. $\dfrac{\sqrt{18}}{\sqrt{2}}$

$\sqrt{\dfrac{18}{2}}$

$\sqrt{9}$

$\boxed{\pm 3}$

l. $\dfrac{\sqrt{24}}{\sqrt{3}}$ $\boxed{= 2\sqrt{2}}$

$\sqrt{\dfrac{24}{3}}$

$\sqrt{8}$

$\sqrt{4 \cdot 2}$

$\sqrt{4} \cdot \sqrt{2}$

m. $\dfrac{6\sqrt{10}}{2\sqrt{2}}$

$\dfrac{6}{2} \sqrt{\dfrac{10}{2}}$

$\boxed{3\sqrt{5}}$

E. So you can multiply and divide with square roots because of the rule of exponents that involves bases with the same exponent. What about addition and subtraction…

1. What is x + x?

$2x$

2. What is 2f + 3f?

$5f$

3. What is 3r – 10r?

$-7r$

4. Why are you allowed to combine the binomials above into a monomial?

combine "like terms"
(same variable with same exp.)

5. What is $\sqrt{2} + \sqrt{2}$?

like x+x except x is $\sqrt{2}$

$2\sqrt{2}$

6. What is $2\sqrt{6} + 3\sqrt{6}$?

like 2f+3f except f is $\sqrt{6}$

$5\sqrt{6}$

7. What is $3\sqrt{11} - 10\sqrt{11}$?

like 3r - 10r except r is $\sqrt{11}$

$-7\sqrt{11}$

8. Dalida claims that $\sqrt{25} + \sqrt{25} = \sqrt{50}$ because 25 plus 25 is 50. Is she right? Explain.

No! $\sqrt{25} = 5$
so $\sqrt{25} + \sqrt{25}$ is 5+5
which is 10 and $\sqrt{50}$ is not 10

$\sqrt{50} \approx 7.1$

$\sqrt{25 \cdot 2}$

$\sqrt{25} \cdot \sqrt{2}$

$5\sqrt{2}$

9. Does x + y simplify? Explain.

No they are different variables

10. Does $\sqrt{7} + \sqrt{10}$ simplify? Explain.

No, they are different bases

You will learn much more about rational (fractional) exponents in algebra II.

$x^{\frac{2}{3}}$ $xy^{3/7}$