

Unit 7B Day 13: Introduction to *Imaginary* numbers!

Focus Question: Are there really no solutions if there are no x-intercepts?

A. Review

1. What were the other names for x intercepts? Explain why they have these names.

roots
Solutions
zeros

2. A linear function always has 1 x intercepts (or solutions) and it is degree 1.

3. A quadratic can have 0, 1, or 2 x-intercepts.

4. A quadratic is degree 2.

$y = mx + b$
 $f(x) = a(x-h)^2 + k$

B. The Fundamental Theorem of Algebra

1. What does fundamental mean?

basic & necessary

2. **The fundamental theorem of algebra: A polynomial of degree n will have exactly n roots.**

So if a quadratic is degree 2, it has to have 2 roots.

3. Graph and solve each of the following in a different color

a. $f(x) = x^2 - 4$

$0 = x^2 - 4$
 $+4 \quad +4$
 $\sqrt{4} = \sqrt{x^2}$
 $x = \pm 2$
 $(2,0) (-2,0)$

When a quadratic has two x intercepts we say it has two real solutions.

b. $g(x) = (x-4)^2$

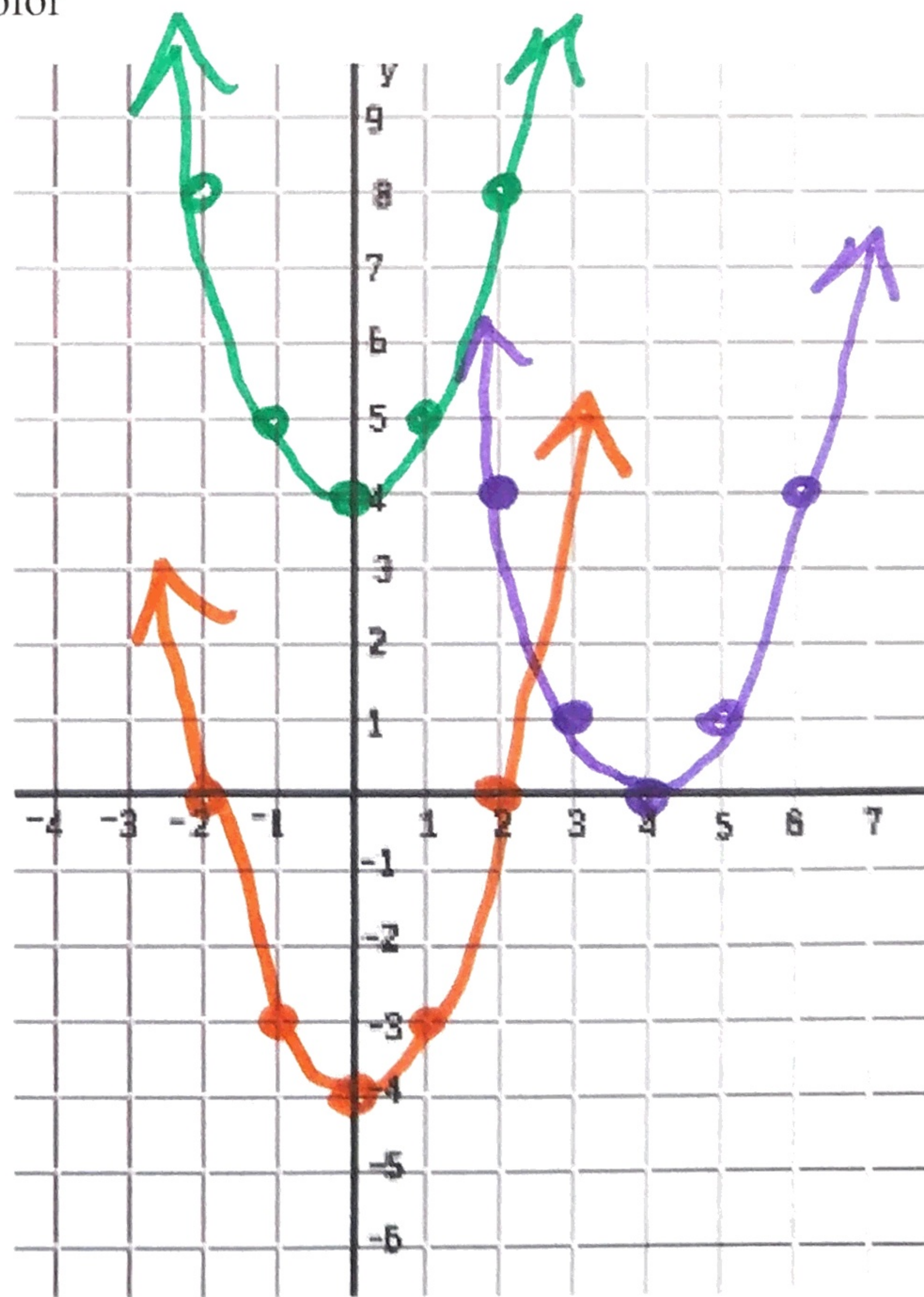
$\sqrt{0} = \sqrt{(x-4)^2}$
 $0 = x - 4$
 $+4 \quad +4$
 $x = 4$
 $(4,0)$

When a quadratic has one x intercept, we say it has one real (repeated) solution and it is always the vertex

c. $h(x) = x^2 + 4$

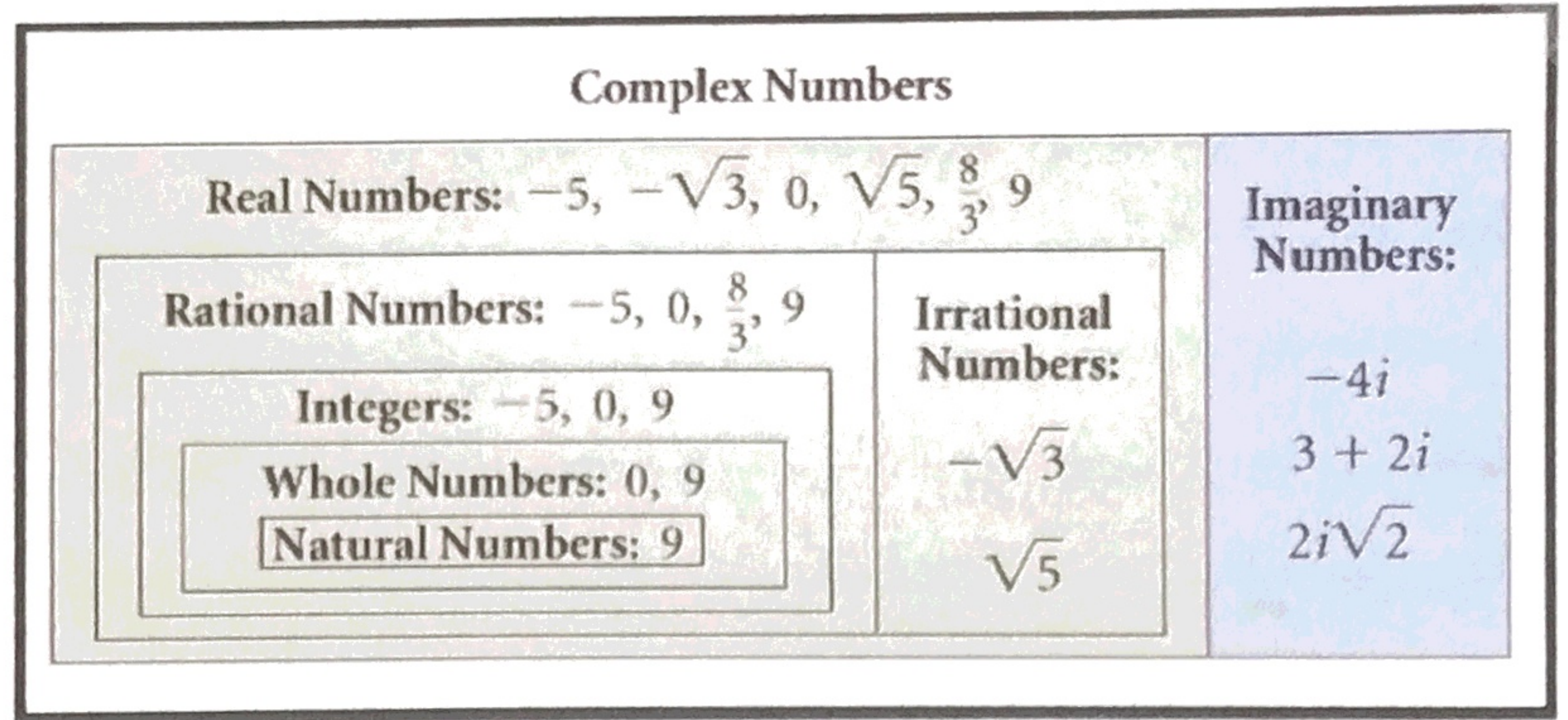
$0 = x^2 + 4$
 $-4 \quad -4$
 $\sqrt{-4} = \sqrt{x^2}$
 $x = \sqrt{-4}$

When a quadratic has no x intercepts we say it has no real solutions. But that doesn't mean that it doesn't have solutions!



C. The complex number system

When a quadratic has no x intercepts, we say it has no real solutions or two complex solutions.



They are called complex solutions because they involve imaginary numbers. See video for intro to imaginary numbers. <http://viewpure.com/T647CGsuOVU?start=0&end=0>

The person who "created" imaginary numbers felt they should be called lateral.

Have you ever seen something like this?

Mathematicians find the humor because

$\sqrt{-1} = i$ so it says i ♥ Math



So what is $\sqrt{-4}$?

$$\sqrt{-4} = \sqrt{4 \cdot -1} = \sqrt{4} \cdot \sqrt{-1} = \pm 2i$$

Simplify each square root.

1. $\sqrt{-25}$

$$\sqrt{25 \cdot -1}$$

$$\sqrt{25} \cdot \sqrt{-1}$$

$$\pm 5i$$

2. $\sqrt{-121}$

$$\sqrt{121 \cdot -1}$$

$$\pm 11i$$

3. $\sqrt{-81}$

$$\sqrt{-81}$$

$$\sqrt{81 \cdot -1}$$

$$\pm 9i$$

4. $\sqrt{-225}$

$$\sqrt{225 \cdot -1}$$

$$\pm 15i$$

5. $\sqrt{-37}$

$$\sqrt{-1 \cdot 37}$$

$$i\sqrt{37}$$

6. $\sqrt{-2}$

$$\sqrt{-1 \cdot 2}$$

$$i\sqrt{2}$$

7. $\sqrt{-105}$

$$\sqrt{-1 \cdot 105}$$

$$i\sqrt{105}$$

8. $\sqrt{-22}$

$$\sqrt{-1 \cdot 22}$$

$$i\sqrt{22}$$

9. $\sqrt{40} \sqrt{-40}$

$$\sqrt{4 \cdot 10 \cdot -1}$$

$$\sqrt{4} \sqrt{-1} \sqrt{10}$$

$$\pm 2i\sqrt{10}$$

10. $\sqrt{300} \sqrt{-300}$

$$\sqrt{100 \cdot -1 \cdot 3}$$

$$\pm 10i\sqrt{3}$$

11. $\sqrt{147} \sqrt{-147}$

$$\sqrt{49 \cdot -1 \cdot 3}$$

$$\pm 7i\sqrt{3}$$

12. $\sqrt{80} \sqrt{-80}$

$$\sqrt{4 \cdot -1 \cdot 20}$$

$$\pm 2i\sqrt{20}$$

$$\pm 2i\sqrt{4 \cdot 5}$$

$$\pm 2i \cdot 2\sqrt{5}$$

$$\pm 4i\sqrt{5}$$