

Unit 7B Day 22: Standard Form back to Vertex Form

Focus Question: If I can go from vertex form to standard form, can I go from standard form to vertex?

A. Completing the Square

The process to turn standard form into vertex form is called completing the square. We are going to start this process by examining perfect square trinomials.

1. List some perfect squares that you already know.

16 100 225 $4x^2$ $(2x)^2$
 $25y^2$

(These are all monomials)

2. Are there perfect square binomials? (Remember "perfect square" means something times itself can make the polynomial.)

No!

~~$x^2 + 49 = (x+7)^2$~~ $\rightarrow x^2 + 14x + 49$
 $(x+7)(x+7)$ \uparrow \uparrow \uparrow
 1 2 3

3. Turn each square into standard form.

a. $(x+6)^2$

$(x+6)(x+6)$
 $x(x+6) + 6(x+6)$
 $x^2 + 6x + 6x + 36$
 $x^2 + 12x + 36$

b. $(x-8)^2$

x	-8
x^2	$-8x$
$-8x$	$+64$

$x^2 - 16x + 64$

c. $(2x+5)^2$

$(2x+5)(2x+5)$
 $2x(2x+5) + 5(2x+5)$
 $4x^2 + 10x + 10x + 25$
 $4x^2 + 20x + 25$
 $(2x)^2$

d. $(4x-3)^2$

$4x$	-3
$16x^2$	$-12x$
$-12x$	$+9$

$16x^2 - 24x + 9$
 $(4x)^2$

e. What do you notice about the a and c terms of the standard form?

a & c are positive perfect squares

f. What pattern do you notice in the b term?

its twice the "a term" times the "c term"

g. Turn $(x-h)^2$ into standard form.

$x^2 - 2xh + h^2$

vertex form \rightarrow

B. Perfect square Trinomials

For a trinomial to be a perfect square, the a and c terms must be a positive perfect squares and the b term must be twice the product of the numbers that were squared.

1. Decide if each trinomial is a perfect square trinomial

a. $x^2 + 6x + 9$

$a = 1$ or 1^2
 $c = 9$ or 3^2
 $2 \cdot 1 \cdot 3 = 6$
Yes!

b. $x^2 - 4x + 16$

$a = 1$ or 1^2
 $c = 16$ or 4^2
 $2 \cdot 1 \cdot 4 = 8$
No!

c. $4x^2 - 4x - 1$

$a = 4$ or 2^2
 $c = -1$
No!

d. $9x^2 - 12x + 4$

$a = 9$ or 3^2
 $c = 4$ or 2^2
 $2 \cdot 3 \cdot (-2) = -12$
Yes

Turning standard form into vertex form...

Review

1) How do you find the vertex from standard form?

find the a.o.s. $x = \frac{-b}{2a}$

then subs. x value into function

2) What is vertex form?

$$f(x) = a(x-h)^2 + k$$

3) What is the a.o.s. from vertex form

$$x = h$$

↓
 $\frac{b}{2a}$

Another way to see this....

How did we check the positive perfect square?

$$\frac{2 \cdot a \cdot c}{2a} = \frac{b}{2a}$$
$$c = \frac{b}{2a}$$

2. Decide if each trinomial is a perfect square trinomial. If it is, write it as a binomial squared.

$a^2 + 4a + 4$ $a=1$ or 1^2
 $c=4$ or 2^2
 $2 \cdot 1 \cdot 2 = 4$
 Yes!
 $(a+2)^2$

$y^2 - 8y + 10$ $a=1$ or 1^2
 $c=10$ or $(\sqrt{10})^2$
 No!

$n^2 - 8n + 16$ $a=1$ or 1^2
 $c=16$ or 4^2
 $2 \cdot 1 \cdot 4 = 8$
 Yes!
 $(n-4)^2$

$x^2 - 10x - 100$ $a=1$ or 1^2
 $c=-100$
 No!

$4x^2 - 4x + 1$ $a=4$ or 2^2
 $c=1$ or $(-1)^2$
 $2 \cdot 2 \cdot (-1) = -4$
 Yes!
 $(2x-1)^2$

$x^2 + 6x - 9$ $a=1$ or 1^2
 $c=-9$
 No!

$n^2 - 13n + 36$ $a=1$ or 1^2
 $c=36$ or 6^2
 $2 \cdot 1 \cdot 6 = 12$
 No!

$9b^2 - 6b + 1$ $a=9$ or 3^2
 $c=1$ or $(-1)^2$
 $2 \cdot 3 \cdot (-1) = -6$
 Yes!
 $(3b-1)^2$

$121y^2 + 22y + 1$ $a=121$ or 11^2
 $c=1$ or 1^2
 $2 \cdot 11 \cdot 1 = 22$
 Yes!
 $(11y+1)^2$

insert notes

4. Show your work to decide what number would need to go in the blank to make a perfect square trinomial. Then write the binomial that was squared.

$x^2 + 16x + \underline{64}$ $\frac{b}{2a}$
 $a=1$ or 1^2
 $b=16$
 $\frac{16}{2(1)} = 8$
 $c=64$ or 8^2
 $(x+8)^2$

$x^2 - 20x + \underline{100}$
 $a=1$ or 1^2
 $b=-20$
 $\frac{-20}{2(1)} = -10$
 $c=100$ or $(-10)^2$
 $(x-10)^2$

$x^2 - 8x + \underline{16}$ $\frac{-8}{2(1)} = -4$
 $a=1$ or 1^2
 $b=-8$
 $c=16$ or $(-4)^2$
 $(x-4)^2$

$x^2 + 50x + \underline{625}$ $\frac{50}{2(1)} = 25$
 $a=1$ or 1^2
 $b=50$
 $c=625$ or $(25)^2$
 $(x+25)^2$

5. What you did in #3 is called completing the square. Why do you think this name is appropriate?

