Name: ______________________ Date: ______________________ Hour: ____Alg 1____

Unit 7B Day 23: Solving by Completing the Square

Focus Question: Where did the quadratic formula come from?

A. All quadratics can be solved by the quadratic formula which is : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

All quadratics have a vertex so ALL quadratics can be solved by "completing the square." To most people, this process is a simpler alternative to the quadratic formula. (There is one more alternative to the quadratic formula that we will learn in the next part, but it does NOT work for ALL quadratics.)

B. Solve each of the following by completing the square. Remember to complete the square you want the *a* and *b* terms to be alone on the same side (Also tell what your answer represents.)

1. $x^2 - 2x - 15 = 0$ — Quad = 0, so x int.

$+15 \quad +15$

$x^2 - 2x + \underline{1} = 15 + \underline{1}$

$\sqrt{(x-1)^2} = \sqrt{16}$

$x - 1 = \pm 4$

$x - 1 = 4$ or $x - 1 = -4$

$+1 \quad +1 \qquad +1 \quad +1$

$x = 5$ $\quad x = -3$

a = 1 or 1^2, b = −2, $\frac{-2}{2(1)}$, c = 1 or $(-1)^2$

2. $x^2 + 6x = 35$ — Quad = const, y = 35

$x^2 + 6x + \underline{9} = 35 + \underline{9}$

$\sqrt{(x+3)^2} = \sqrt{44}$ (4·11)

$x + 3 = \pm 2\sqrt{11}$

$-3 \qquad -3$

$x = -3 \pm 2\sqrt{11}$ exact

$x = -3 + 2\sqrt{11}$ or $x = -3 - 2\sqrt{11}$

$x \approx 3.63$ or $x \approx -9.63$ approx.

a = 1 or 1^2, b = 6, c = 9 or 3^2, $\frac{6}{2(1)}$

3. $x^2 - 4x - 8 = 0$ — Quad = 0, x int.

$+8 \quad +8$

$x^2 - 4x + \underline{4} = 8 + \underline{4}$

$\sqrt{(x-2)^2} = \sqrt{12}$

$x - 2 = \pm\sqrt{12}$ (4·3)

$x - 2 = \pm 2\sqrt{3}$

$+2 \qquad +2$

$x = 2 \pm 2\sqrt{3}$ exact

$x = 2 + 2\sqrt{3}$ $\quad x = 2 - 2\sqrt{3}$

$x \approx$ $\quad x \approx$

a = 1 or 1^2, b = −4, c = 4 or $(-2)^2$, $\frac{-4}{2(1)} = -2$

4. $x^2 - 7x = 18$ — Quad = const, y = 18

$x^2 - 7x + \frac{49}{4} = 18 + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{72}{4} + \frac{49}{4}$

$\sqrt{\left(x - \frac{7}{2}\right)^2} = \sqrt{\frac{121}{4}}$

$x - \frac{7}{2} = \pm\frac{11}{2}$

$x - \frac{7}{2} = \frac{11}{2}$ or $x - \frac{7}{2} = -\frac{11}{2}$

$+7/2 \quad +7/2 \qquad +7/2 \quad +7/2$

$x = 9$ $\quad x = -2$

a = 1 or 1^2, b = −7, c = $\frac{49}{4}$ or $\left(-\frac{7}{2}\right)^2$, $\frac{-7}{(2)(1)}$

Quad ↙ const ↙  $y=-2$

5. $2x^2+8x-6=-2$

$+6 \quad +6$

$2x^2+8x=4$

$\frac{2x^2+8x}{2} = \frac{4}{2}$

$x^2+4x+\underline{4}=2+\underline{4}$

$\sqrt{(x+2)^2}=\sqrt{6}$

$x+2=\pm\sqrt{6}$

$-2 \quad -2$

$\boxed{x=-2\pm\sqrt{6}}$ exact

$a=1 \text{ or } 1^2$
$b=4$
$c=4 \text{ or } 2^2$

$\frac{4}{2(1)}=2$

$x\approx -2+\sqrt{6} \approx 0.45$

$x\approx -2-\sqrt{6} \approx -4.45$

Line & 

6. $8x = 4x^2-1$

$-8x \quad -8x+1$

$+1$

$1+4=4x^2-8x+4$

$\sqrt{5}=\sqrt{(2x-2)^2}$

$2x-2=\pm\sqrt{5}$

$+2 \quad +2$

$\frac{2x}{2}=\frac{2\pm\sqrt{5}}{2}$

$\boxed{x=\frac{2\pm\sqrt{5}}{2}}$ exact

$x\approx 2.12$

$x\approx -0.12$

$a=4 \text{ or } 2^2$
$b=-8$
$c=\text{or}_4\ (-2)^2$

$\frac{-8}{2(2)}=-2$

7. $2x^2-16x+5=-5$

$-5 \quad -5$

$\frac{2x^2-16x}{2} = \frac{-10}{2}$

$a=1 \text{ or } 1^2$
$b=-8$
$\frac{-8}{2(1)}=-4$
$c=(-4)^2 \text{ or } 16$

$x^2-8x+\underline{16}=-5+\underline{16}$

$\sqrt{(x-4)^2}=\sqrt{11}$

$x-4=\pm\sqrt{11}$

$+4 \quad +4$

$\boxed{x=4\pm\sqrt{11}}$

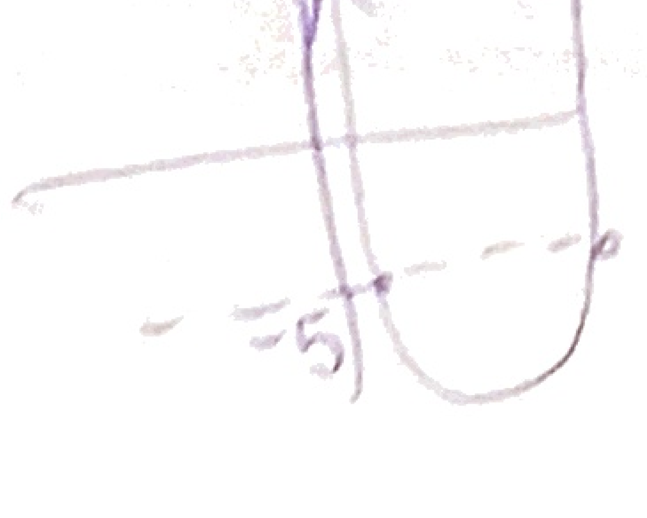
exact

$x\approx 4+\sqrt{11} \rightarrow 7.32$

$x\approx 4-\sqrt{11} \rightarrow 0.68$

approx

Where parabola intersects constant



8. $ax^2+bx+c=0$

$-c \quad -c$

$\frac{ax^2+bx}{a}=\frac{-c}{a}$

$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{-c}{a}+\frac{b^2}{4a^2}$

$a=1 \text{ or } 1^2$
$b=\frac{b}{a}$
$c=\frac{b^2}{4a^2} \text{ or } \left(\frac{b}{2a}\right)^2$

$\frac{\frac{b}{a}}{2(1)}$

$\frac{b}{a}\cdot\frac{1}{2}=\frac{b}{2a}$

$\left(x+\frac{b}{2a}\right)^2=\frac{-c(4a)}{4a^2}+\frac{b^2}{4a^2}$

$=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}-\frac{b}{2a}$

$-\frac{b}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$