

Unit 8 Day 2: The Exponential Family

Focus Question: Can I understand the equation for the exponential family?

A. Exponential Function Equations

1. When Joe asked for his new allowance, the equation for the pennies received under his suggested plan is $f(x) = \frac{1}{2} \cdot 2^x$. How can you tell by looking that this is not a linear or a quadratic function?

The exponent is not 1 (linear) nor 2 (quad).
The exponent is x .

The doubles penny plan is an example of an exponential function.

2. Look back at the equation, what part of the equation tells you that it is exponential?

Degree is x

3. What part of the equation tells you that this is a doubling plan?

the base is 2 & $\cdot 2$ means double

4. Where could the $\frac{1}{2}$ be seen in the situation? (Hint: look at the table

and it is a very important part of linear situations as well.)

$\frac{1}{2}$ is the yint.

Week	# Pennies
1	1 $\cdot 2$
2	2 $\cdot 2$
3	4 $\cdot 2$
4	8
5	16

Just like all linear functions can be written as $f(x) = mx + b$, all exponential functions can be written as $f(x) = a \cdot b^x$

5. Just like in linear, x still stands for input or independent variable.
6. Just like in linear, $f(x)$ or y still stands for output or dependent variable.
7. But don't let b fool you! It does NOT stand for y-intercept! In exponential b stands for base which is the number that is constantly being multiplied. In linear, the rate of change (or slope or m) is additive (the same number is always added). The rate in exponential functions is called multiplicative because it is constantly being multiplied. Because it is being multiplied another word for it is factor. It is NOT called the slope.
8. The a in an exponential function is called the initial value. Another word for this is y-intercept. Just like linear, this still occurs when the x value is 0.
9. For each equation below, give the y intercept and the base.

$$f(x) = \frac{1}{3} \cdot 6^x$$

base is 6
yint $(0, \frac{1}{3})$

$$f(n) = 10 \cdot 2^n$$

base is 2
yint $(0, 10)$

$$f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$$

base $\frac{1}{2}$
yint $(0, 4)$
initial value

GEMS

10. Evaluate each function for $f(3)$ and $f(-2)$

$$f(x) = \frac{1}{3} \cdot 6^x$$

$$\begin{aligned} f(3) &= \frac{1}{3} \cdot 6^3 \\ &= \frac{1}{3} \cdot 216 \\ &= 72 \end{aligned}$$

$(3, 72)$

$$\begin{aligned} f(-2) &= \frac{1}{3} \cdot 6^{-2} \\ &= \frac{1}{3} \cdot \frac{1}{6^2} \\ &= \frac{1}{3} \cdot \frac{1}{36} \\ &= \frac{1}{108} \end{aligned}$$

$(-2, \frac{1}{108})$

$$\begin{aligned} f(n) &= 10 \cdot 2^n \\ f(3) &= 10 \cdot 2^3 \\ &= 10 \cdot 8 \\ &= 80 \end{aligned}$$

$(3, 80)$

$$\begin{aligned} f(-2) &= 10 \cdot 2^{-2} \\ &= \frac{10}{2^2} \Rightarrow \frac{10}{4} \\ &= \frac{5}{2} \end{aligned}$$

$(-2, \frac{5}{2})$

$$f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$$

$$\begin{aligned} f(3) &= 4 \cdot \left(\frac{1}{2}\right)^3 \\ &= 4 \cdot \left(\frac{1^3}{2^3}\right) \\ &= 4 \cdot \left(\frac{1}{8}\right) = \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

$(3, \frac{1}{2})$

$$\begin{aligned} f(-2) &= 4 \cdot \left(\frac{1}{2}\right)^{-2} \\ &= 4 \cdot 2^2 \\ &= 4 \cdot 4 \\ &= 16 \end{aligned}$$

$(-2, 16)$

B. The parent exponential function is $f(x) = 1 \cdot 2^x$ but is typically written $f(x) = 2^x$

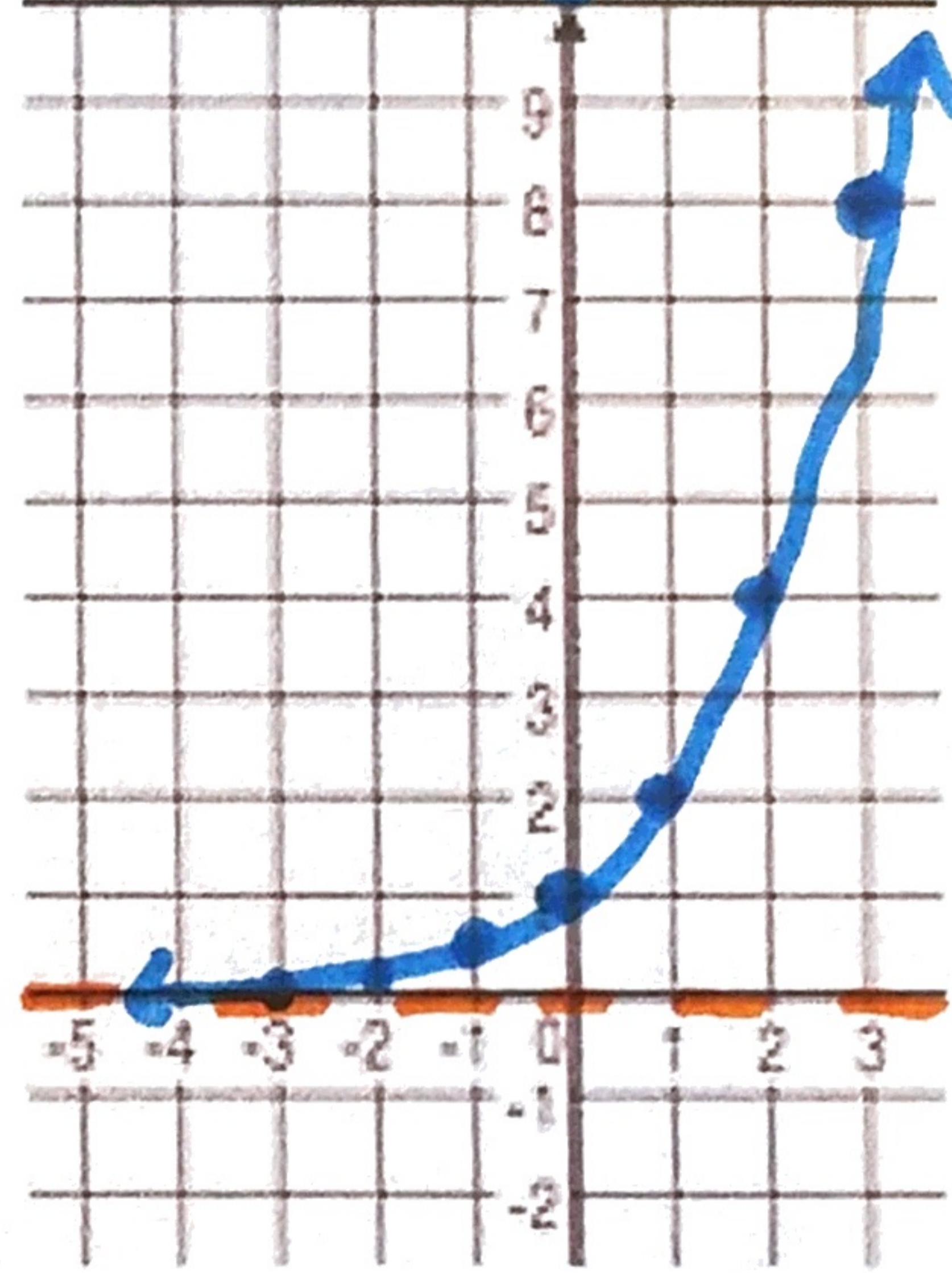
1. Why do they leave off the 1?

1 is implied bc it doesn't change value

2. Complete the table and graph

x	f(x)
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$$\begin{aligned} 2^{-1} &= \frac{1}{2} \\ 2^0 &= 1 \\ 2^1 &= 2 \\ 2^2 &= 4 \\ 2^3 &= 8 \end{aligned}$$



3. Will $f(x)$ ever reach zero?

NO

A term for a value that a function approaches but never actually reaches is called an **asymptote**. This is indicated on a graph with a dashed horizontal line.

A Note:

Exponentials that have been translated (left/right or up/down) can be very difficult to identify/write equations of. For example, our original function for Joe's allowance could also have been written as $f(x) = 2^{x-1}$ due to the rules of exponents.

2^{x-1} (Parent translated 1 unit right)

$\frac{2^x}{2^1}$ (Quotient rule of exponents: when bases are divided you subtract the exponents)

$\frac{2^x}{2}$ (Don't really need the exponent 1 because its implied)

$\frac{1}{2} \cdot 2^x$ (Another way to write divided by 2 is times $\frac{1}{2}$)

For this reason, we will only work with exponentials written using the standard form $f(x) = a \cdot b^x$ (NO translating left or right. We will NOT translate them up or down either!)