

Unit 8 Day 7: Decay rates

Focus Question: How are decay rates different from decay factors?

A. Not all exponential functions grow. The opposite of exponential growth is **exponential decay**. The function will decay if the dependent values constantly decrease by the same factor over a period of time. The factor is now called a decay factor.

1. What was the equation for exponential growth when a growth factor was given? $f(x) = a \cdot b^x$
2. What was the equation for exponential growth when growth rate was given? $f(x) = A \cdot (1+r)^x$

3. There is such a thing as a rate of decay. It is the percent at which something loses value over time. How do you think the equation will be different from the equation for exponential growth rates?

4. The equation for rate of decay is $f(x) = A \cdot (1-r)^x$

B. Decay Factors and Decay rates

1. Find the decay factor associated with each rate of decay.

a. 40%

$$f = 1 - 0.4$$

$$f = 0.6$$

b. 35%

$$f = 1 - 0.35$$

$$f = 0.65$$

c. 90%

$$f = 1 - 0.9$$

$$f = 0.1$$

$$\frac{f}{+r} = \frac{1-r}{+r}$$

$$-f + r = 1 - f$$

$$\underline{\quad \quad \quad}$$

$$r = 1 - f$$

2. Find the rate of decay associated with each decay factor

a. 0.4

$$1 - 0.4 = 0.6$$

$$60\%$$

b. 0.85

$$1 - 0.85 = 0.15$$

$$15\%$$

c. 1/4

$$1 - \frac{1}{4} = \frac{3}{4}$$

$$75\%$$

C. Writing equations of exponential growth and decay

1. A flea medicine breaks down at a rate of 20% per hour. This means that as each hour passes, 20% of the active medicine is used. This is the rate of decay of the medicine. The initial dose is 60 milligrams.

a. What is the equation for how much medicine is in the bloodstream after so many hours?

M : medicine
 h : hours since dose

$$M(h) = 60 \cdot (1 - 0.2)^h = 60 \cdot (0.8)^h$$

b. How much medication will be in the bloodstream after 2 hours?

$$M(2) = 60 \cdot 0.8^2 = 38.4 \text{ mm}$$

c. When will the medication be down to 10 milligrams?

After about 8 hrs.

2. New cars start to depreciate as soon as you drive them off the lot. Assuming you are going to own your car for more than 1 year, cars depreciate (lose value) at a rate of around 18% per year. If you received a Honda accord for your 15th birthday and were going to trade it in when you leave for college at the age of 18, how much can you expect as a trade in value?

$$V(y) = 24020 \cdot (1 - 0.18)^y$$

V : Value
 y : years

$$V(3) = 24020(0.82)^3$$

~~\$~~ \$10859.97

2020 Honda Accord / MSRP

From \$24,020



3. A tree farm has begun to harvest a section of trees that was planted a number of years ago.

Supply of Trees

Year	0	1	2	3	4	5	6	7	8
Trees Remaining	10,000	9,502	9,026	8,574	8,145	7,737	7,350	6,892	6,543

- a. Suppose the relationship between the year and the trees remaining is exponential. Approximate the decay factor for this relationship.

$$\frac{9502}{10000} \quad \boxed{0.9502}$$

- b. At what rate is the tree farm harvesting trees?

$$1 - 0.95 \quad 5\%$$

- c. Write an equation for the relationship between time and trees remaining.

T: trees
y: years

$$T(y) = 10000(0.95)^y$$

- d. When will the number of trees be less than 100?

after 90 but ~~more~~ less 91

4. A half life is the amount of time that it takes for half of a substance to decay. DDT is a pesticide that was widely used until it was banned in 1972 because of its harm to the environment. DDT has a half life of 15 years.

- a. In what year will half of the DDT sprayed in 1972 be gone?

$$\begin{array}{r} + 15 \\ \hline 1987 \end{array}$$

- b. How many half lives have passed since 1972?

$$\begin{array}{r} 2020 \\ - 1972 \\ \hline 48 \end{array} \quad \frac{48}{15} \approx 3 \text{ half lives}$$

- c. If a forest was sprayed with a substance that contained 100grams of DDT, just before it was banned, how much DDT is still in the forest?

$$f(x) = 100 \cdot \left(\frac{1}{2}\right)^x$$

$$f(3) = 100 \cdot \left(\frac{1}{2}\right)^3 = \boxed{12.5 \text{ grams}}$$