Name: ____________________ **Date:** ________________ **HW # 86: Growth and decay**

Cassie's grandmother wanted to help save for Cassie to go to college. She doesn't have much money right now, but found two different options at different banks.

Option 1	OR	Option 2
\$1,000 at 3% interest per year		\$800 at 6% per year
$f(x) = 1000 \cdot (1.03)^x$		$f(x) = 800 \cdot (1.06)^x$

1. If Cassie is currently 2 years old, which option will give her the most money when she goes to college at age 18? Explain. $18-2=16$ She'll save for 16yrs $\boxed{\text{Option 2}}$

   $f(16) = 1000 \cdot (1.03)^{16}$ ≈ 1604.71 $<$ $f(16) = 800 \cdot (1.06)^{16}$ ≈ 2032.28

2. If you have a long time to invest, which is better a higher initial value or a higher rate? option 1 had the higher intial value but lower rate, Option 2 had the lower initial value but higher rate. Option 2 was better for 16yrs to save, so higher rate is better.

3. If Cassie is currently 15, which option will give her the most money when she goes to college at age 18? Explain. $18-15=3$

   $f(3) = 1000 \cdot (1.03)^3$ ≈ 1092.73 $>$ $f(3) = 800 \cdot (1.06)^3$ ≈ 952.81 $\boxed{\text{option 1}}$

4. If you have a short time to invest, which is better, a higher initial value or a higher rate? option 1, the higher initial value was better.

5. A hypothetical strain of bacteria doubles every 5 minutes. One single bacterium was put in a sealed bottle at 9:00Am and the bottle was filled at exactly 10:00 AM. At what time was the bottle one-half full? (*Think in terms of the doubling time.)

   9:55AM If it's full at 10 & it doubles every 5min, it was ½ full, five minutes prior.

6. A computer valued at \$6500 depreciates at the rate of 14.3% per year.

   a. Write a function that models the value of the computer.

   $f(x) = 6500(1-0.143)^x$ or $f(x) = 6500(0.857)^x$

   b. Find the value of the computer after three years.

   $f(3) = 6500 \cdot (0.857)^3$

   ≈ 4091.25

   $\boxed{\$4091.25}$

7. The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%.

a. Find the growth factor for the world population.

factor = $1+r$ $\quad 1+0.0126 = \boxed{1.0126}$

b. Suppose the rate of increase continues to be 1.26% . Write a function to model the world population

$f(x) = 6{,}080{,}000{,}000 \cdot (1.0126)^x$

c. Let x be the number of years past the year 2000. Find the world population in 2010.

$f(10) = 6080000000 \cdot (1.0126)^{10}$

$2010 - 2000 = 10$

$\boxed{\approx 6{,}891{,}008{,}880}$

8. Find a bank account balance if the account starts with \$100, has an annual rate of 4%, and the money left in the account for 12 years.

$f(12) = 100 \cdot (1.04)^{12}$

$\boxed{\approx \$160.10}$

9. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$1994 - 1985 = 9$

$f(9) = 285 \cdot (1.75)^9$

$\boxed{\approx 43872 \text{ people (no decimals!)}}$

10. The population of Winnemucca, Nevada, can be modeled by P=6191(1.04)^t where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

initial value

$\boxed{6{,}191 \text{ people}}$

factor−1

$1.04 - 1 = 0.04$

$\boxed{4\%}$

11. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

$f(51) = 30{,}000(1.05)^{51}$

$2011 - 1960 = 51$

$\boxed{\approx \$361223.09}$

12. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

$f(x) = 500 \cdot (1-.12)^x$ or $\boxed{f(x) = 500(0.88)^x}$

$f(240) = 500(0.88)^{240}$

$\boxed{\approx 2.4 \times 10^{-11} \text{ mL}}$

13. Write an exponential function to model each situation. Find the value of each function after five years.

a. A \$12,500 car depreciates 9% each year

$f(x) = 12500(1-0.09)^x$

$\boxed{f(x) = 12500(0.91)^x}$

$f(5) = 12500(0.91)^5$

$\boxed{\approx \$7800.40}$

b. A baseball card bought for \$50 increases 3% in value each year.

$\boxed{f(x) = 50 \cdot (1.03)^x}$

$f(5) = 50(1.03)^5$

$\boxed{= \$57.96}$