Name: ____________________________________ Date: _________________________ Hour: ____Alg 1____

Unit 8 Day 1: Introduction to Exponential Functions (The Allowance Problem)

Focus Question: Are all functions linear?

A. Review: Use $5 \cdot 2^4$ (which is written in _____________________ form) to answer the following..
   1. What is the base(s)?
   2. What is the exponent(s)?
   3. What do exponents tell you?
   4. Write the number in expanded form.
   5. Show all work using order of operations to write the number in standard form.

B. The Allowance Problem

   Joe's parents usually give him $10 per week for doing his chores. With a new year about to start, Joe went to his parents and asked for the following:

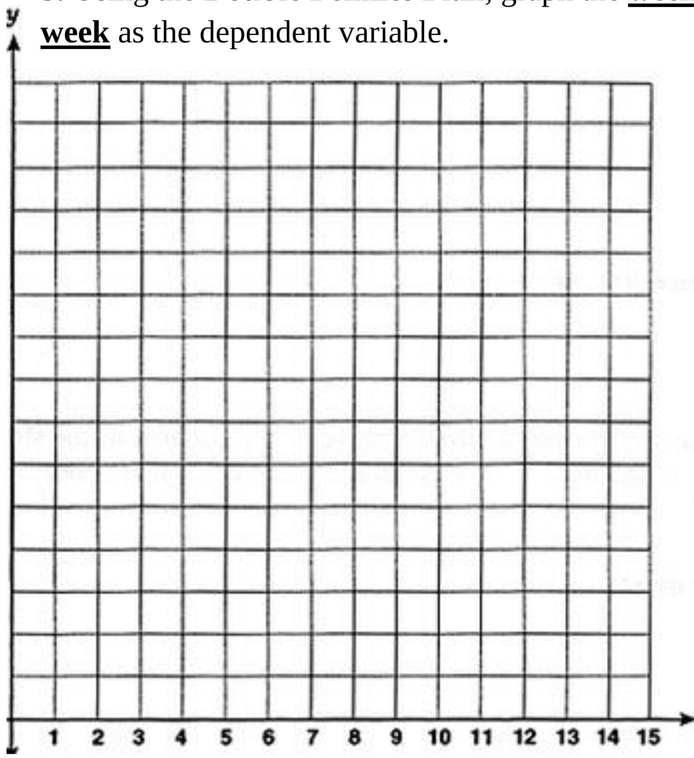
   > "The first week, I would like to receive a penny. The 2nd week I would like to get two pennies. The third week I want to get four pennies. The fourth week I'd get 8 pennies. Every week I'd like to get double the number of pennies from the week before."

   1. Pre-Work Question (1 minute)
      Do you think Joe is brilliant or crazy for asking for this allowance? Explain
   2. Using each plan, complete the table.

$10 Per Week Plan		
Week	$ Received for week	Total $ in the year
1	10	10
2	10	20
3	10	30
4	10	
5		
6		
7		
8		
9		
10		
11		
12		

Double Pennies Plan		
Week	$ Received for week	Total $ in the year
1	0.01	0.01
2	0.02	0.03
3	0.04	0.07
4	0.08	
5	0.16	
6		
7		
8		
9		
10		
11		
12		

3. Using the **Double Pennies Plan**, graph the **<u>weeks</u>** as the independent variable and the **<u>$ Received for the week</u>** as the dependent variable.

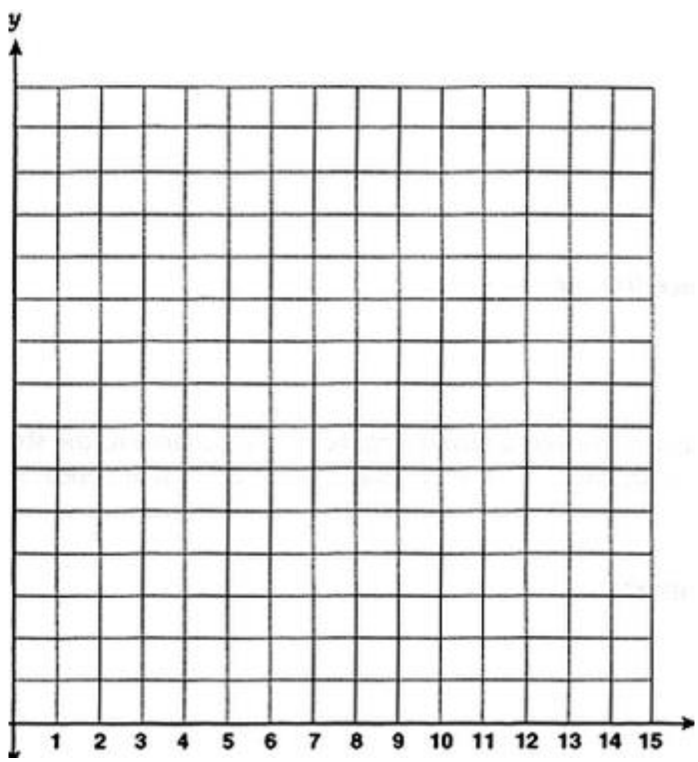


4. Is this graph linear or non-linear? Explain.

5. Is this graph a member of the quadratic family? Explain.

6. On what week will Joe receive about $10 under this plan?

7. Using the **Double Pennies Plan**, graph the **<u>weeks</u>** as the independent variable and the **<u>TOTAL $ Received</u>** as the dependent variable.



8. Is this graph linear or non-linear? Explain.

9. Is this graph a member of the quadratic family? Explain

10. On what week will Joe have close to $520 (which was the total amount received on the $10 per week plan)?

11. Does it belong to the same family as the previous graph? Explain.

C. Summary Questions

1) If you were Joe's parents, **would you agree to his request**? You should support your answer with mathematics.

2) When you look at the tables for the Double Pennies plan (which are below in #3 so you don't have to look back), how can you tell that the tables are not linear and not quadratic?

3) Is there still a pattern to the tables? Explain how the pattern is different from a linear table.

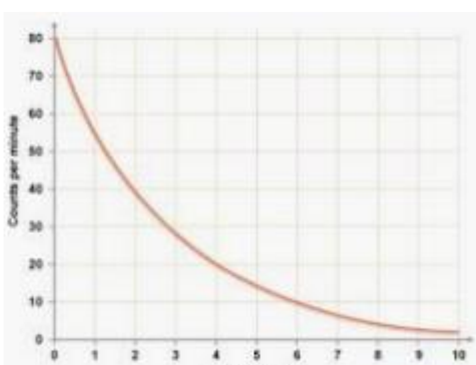
Week	# Pennies
1	1
2	2
3	4
4	8
5	16

Week	Total Pennies
1	1
2	3
3	7
4	15
5	31

4) How would you describe the shape of the graph of the money received on the double pennies plan? (Do **not** just say non-linear!)

These two graphs and tables belong to what is called the **<u>exponential family</u>**.

- You can table a table is exponential if ____
- You can tell a graph is exponential if ____ (Just like a parabola can flip upside down and still be a quadratic, a J can flip backwards and still be exponential)



Name: ___________________________________ Date: _________________________ Hour: ____Alg 1____

Unit 8 Day 2: The Exponential Family

Focus Question: Can I understand the equation for the exponential family?

A. Exponential Function Equations

1. When Joe asked for his new allowance, the equation for the pennies received under his suggested plan is $f(x) = \frac{1}{2} \cdot 2^x$. How can you tell by looking that this is not a linear or a quadratic function?

The doubles penny plan is an example of an **exponential function**.

2. Look back at the equation, what part of the equation tells you that it is exponential?

3. What part of the equation tells you that this is a *doubling* plan?

4. Where could the $\frac{1}{2}$ be seen in the situation? (Hint: look at the table and it is a very important part of linear situations as well.)

Week	# Pennies
1	1
2	2
3	4
4	8
5	16

Just like all linear functions can be written as *f(x) = mx + b*, **all exponential functions can be written as** $f(x) = a \cdot b^x$

5. Just like in linear, *x* still stands for _______________ or _____________________ variable.
6. Just like in linear, *f(x)* or *y* still stands for _______________ or _____________________ variable.
7. But don't let *b* fool you! It does NOT stand for y-intercept! **In exponential *b* stands for ____________ which is the number that is constantly being multiplied.** In linear, the rate of change (or slope or *m*) is additive (the same number is always added). The rate in exponential functions is called multiplicative because it is constantly being multiplied. Because it is being multiplied another word for it is _______________. **It is NOT called the slope**.
8. The *a* in an exponential function is called the **initial value**. Another word for this is ___-___________________. Just like linear, this still occurs when the ____ value is _______.
9. For each equation below, give the y intercept and the base.

$f(x) = \frac{1}{3} \cdot 6^x$ $\qquad$ $f(n) = 10 \cdot 2^n$ $\qquad$ $f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$

10. Evaluate each function for f(3) and f (-2)

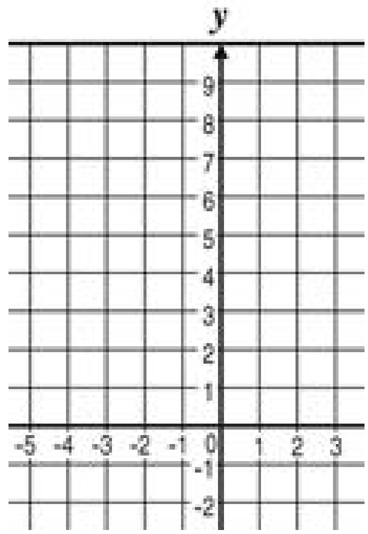
$f(x) = \frac{1}{3} \cdot 6^x$ $\qquad$ $f(n) = 10 \cdot 2^n$ $\qquad$ $f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$

B. The parent exponential function is $f(x) = 1 \cdot 2^x$ but is typically written $f(x) = 2^x$

1. Why do they leave off the 1?

2. Complete the table and graph

x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	



3. Will f(x) ever reach zero? _______

A term for a value that a function approaches but never actually reaches is called an **asymptote**. This is indicated on a graph with a dashed horizontal line.

A Note:
Exponentials that have been translated (left/right or up/down) can be very difficult to identify/write equations of. For example, our original function for Joe's allowance could also have been written as $f(x) = 2^{x-1}$ due to the rules of exponents.

2^{x-1} (Parent translated 1 unit right)

$\frac{2^x}{2^1}$ (Quotient rule of exponents: when bases are divided you subtract the exponents)

$\frac{2^x}{2}$ (Don't really need the exponent 1 because its implied)

$\frac{1}{2} \cdot 2^x$ (Another way to write divided by 2 is times ½)

For this reason, we will only work with exponentials written using the standard form $f(x) = a \cdot b^x$ (NO translating left or right. We will NOT translate them up or down either!)

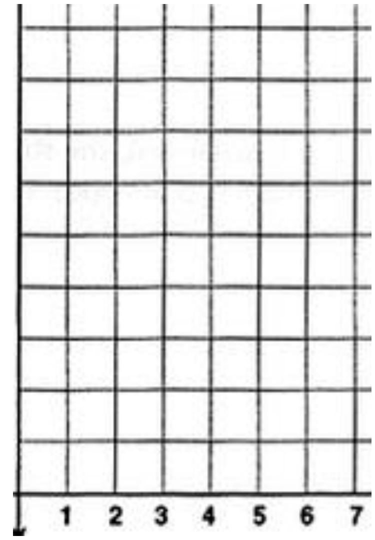
Name: ___________________________________ Date: _________________________ Hour: ___Alg 1____

Unit 8 Day 3: Exponential Growth and Decay

Focus Question: How can I identify whether a function is growing or decaying?

A. Making Ballots
Glen is the secretary of the Student Government Association. He is making ballots for a upcoming school vote. He started with one piece of paper and then started cutting the papers in half to create more ballots.

Cuts	Ballots
0	1
1	2
2	4
3	
4	
5	
6	
7	



1. Complete the table and graph to show the number of ballots after each of the first 7 cuts.
2. Explain how you can tell this is exponential.
3. Using your knowledge of the y intercept and the rate, what is the equation for the function?
4. When you look at the equation, what is the initial value? ______ What is the base? ______
5. Is the number of ballots getting larger or getting smaller?

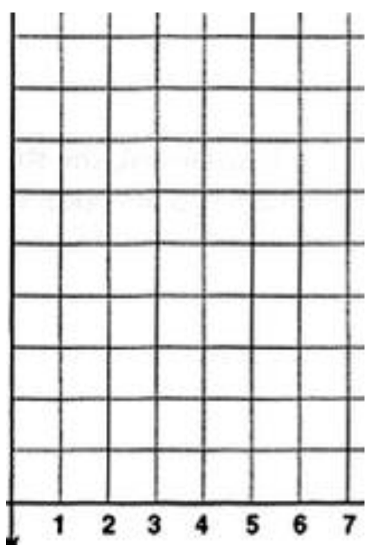
   The term for this is _____________________________________.

   Which part (the initial value or the base) do you think determines it? Explain.

B. Ballot Size
When Glen started making ballots, his paper was 64 square inches.
As he cuts the paper in half, each ballot gets smaller and smaller.

1. Fill in just the table and graph after the first 5 cuts.

Cuts (c)	Size z in sq. inches **(Standard Form)**
0	64
1	32
2	16
3	
4	
5	



2. How can you tell the function is exponential?
3. What is the initial value?
4. What is the base?
5. What is the equation?
6. Is the size getting larger or smaller?

The term for this is _____________________________________. Which part, the initial value or the base, do you think determines it?

C. Exponential Growth and Decay

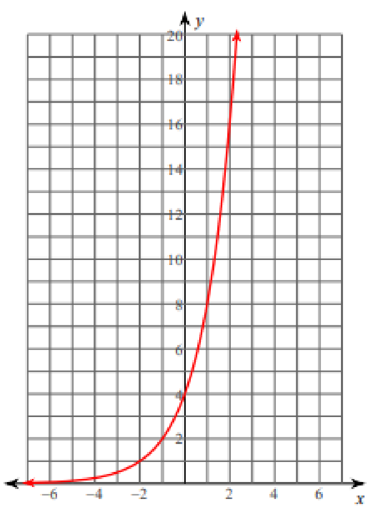
An exponential function is **growth** if the factor (or base) is ________________________________.

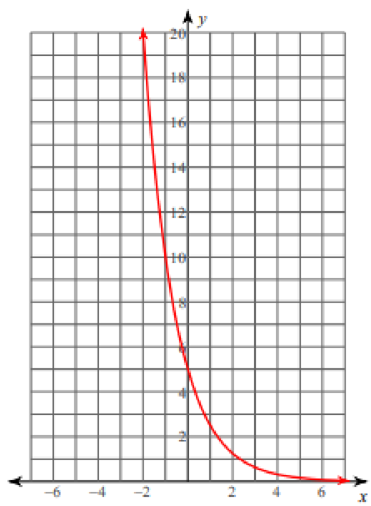
An exponential function is **decay** if the factor (or base) is ________________________________.

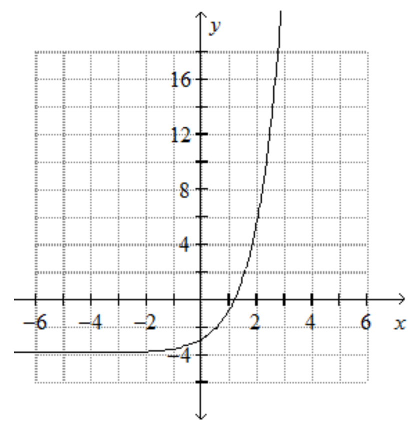
Identify if each equation below represents growth or decay. Explain your answer.

1) $y=\frac{1}{2}\cdot 5^{x}$ 2) $y=4\cdot\left(\frac{2}{3}\right)^{x}$ 3) $y=2\cdot\left(\frac{8}{5}\right)^{x}$ 4) $y=\frac{4}{3}\cdot\left(\frac{7}{10}\right)^{x}$

D. Decide if each graph is exponential growth or decay. Give the equation of the asymptote.



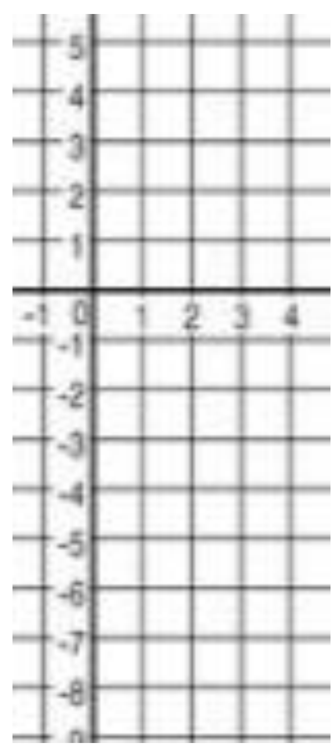




E. Can exponential functions have a negative base?

Fill in the table and graph for $y=1\cdot(-2)^{x}$

Input (x)	Equation looks like	Math looks like	Output (y)
0			
1			
2			
3			



Name: ___________________________________ Date: _________________________ Hour: __Alg 1__

Unit 8 Day 4: Writing and Graphing Exponential Functions

Focus Question: Can I transfer between an equation, table, and graph of an exponential function?

A. Graphing Exponential functions

For each function, identify if it is growth or decay. Then, fill in the table of values and graph. Finally, answer the questions.

1. $f(x) = 3 \cdot 2^x$

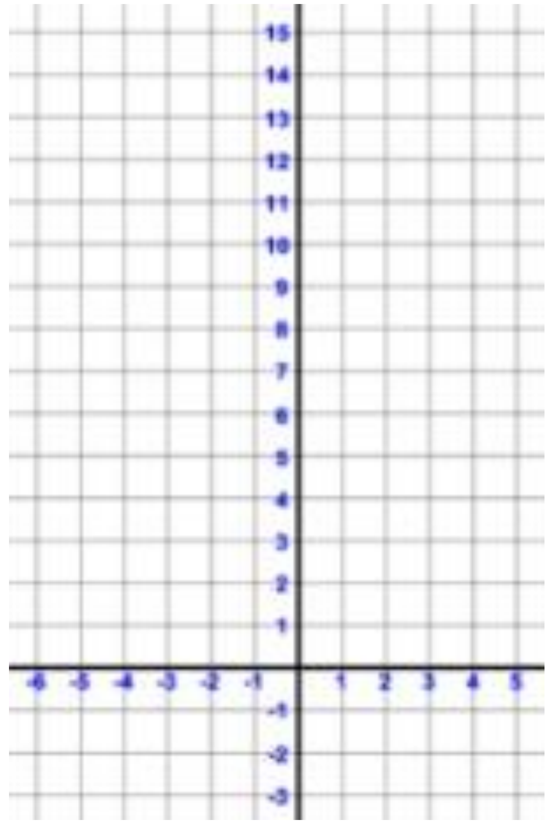
x	f(x)
-2	
-1	
0	
1	
2	

This is exponential _________

because ______________

What is the equation of the asymptote?

What is the range?



2. $g(x) = 6 \cdot \left(\frac{1}{3}\right)^x$

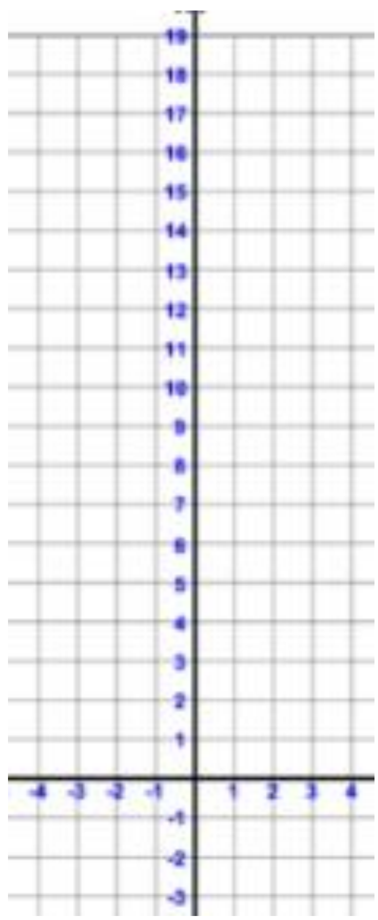
x	g(x)
-1	
0	
1	
2	
3	

This is exponential ___________

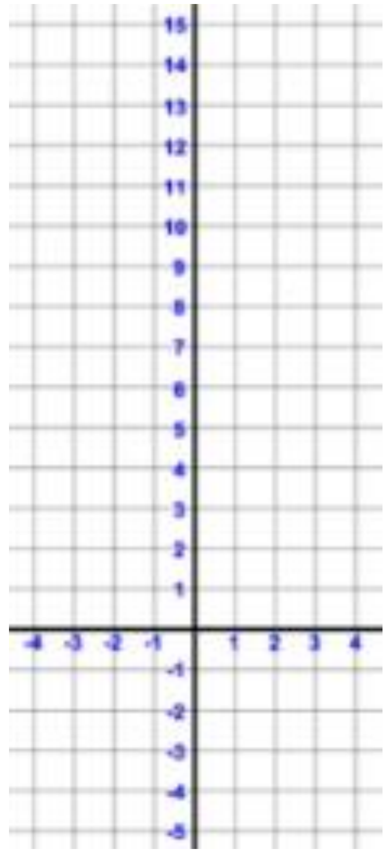
because ___________________

What is the equation of the asymptote?

What is the domain?



3. $j(x) = 4 \cdot \left(\frac{2}{3}\right)^x$



x	j(x)
-1	
0	
1	
2	
3	

This is exponential _____________

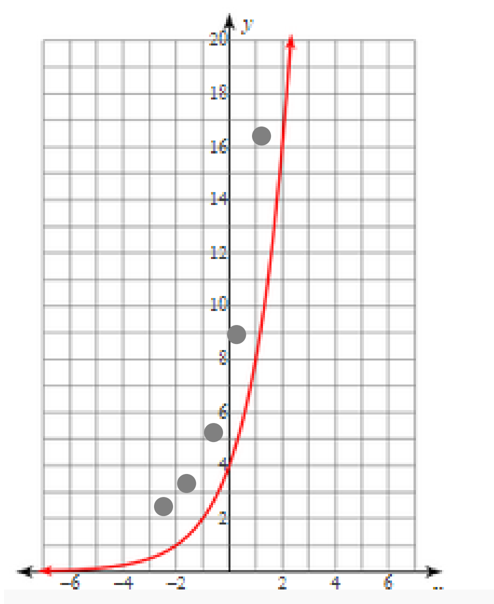
because ____________________________.

B. Graphs to equations

Just like making a table was a good step when going from the equation to the graph, it is good to make a table when going from a graph to an equation.

For each graph below, make a table and then give the equation.

1.



x	f(x)

The equation is

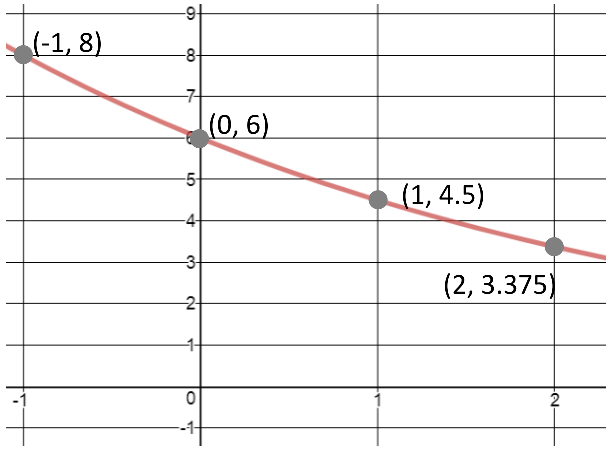
____

*To find out what you are multiplying by, you actually do ____________________

For example: 16/8 = 2 and 8/4 = 2 and 4/2 = 2 and 2/1 = 2

This is an important note b/c it is NOT always obvious what you are multiplying by!

2.

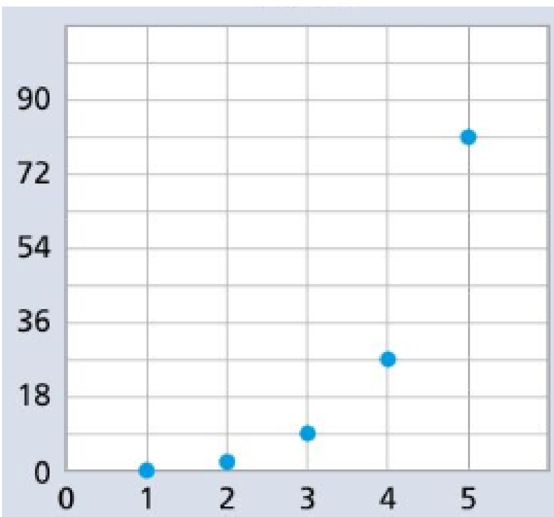


x	f(x)

The equation is

____

3.



x	f(x)

The equation is

____

Name: ______________________________________ Date: ___________________________ Hour: __Alg 1__

Unit 8 Day 5: Writing and Interpreting Exponential Functions

Focus Question: What does the equation of an exponential function mean and how do I use it?

A. Students of Magnolia Middle School conducted an experiment. They put a mixture of chicken bouillon (BOOL yahn), gelatin, and water in a shallow pan. Then they left it out to mold. Each day, the students recorded the area of the mold in square millimeters.

The students wrote the equation $m(d) = 50 \cdot 3^d$ to model the growth of the mold. In this equation, $\boldsymbol{m}$ is the area of the mold in square millimeters after $\boldsymbol{d}$ days.

1. What is the area of the mold at the start of the experiment?

2. What is the growth factor?

3. What is the area of the mold after 5 days?

4. On which day will the area of the mold reach 6,400 mm^2? (Hint, use the table feature on your calculator).

B. If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose a single bacterium lands on your tooth and starts multplying by a factor of 4 every hour.

1. Write an equation that describes the number of bacteria, b, in the new colony after h hours.

2. How many bacteria will be in the colony after 7 hours? ______________________________

3. How many bacteria will be in the colony after 8 hours? ______________________________

4. How can you use the answer from #2 to find the answer to #3 instead of using the equation?

5. After how many hours will there be at least 1,000,000 bacteria in the colony?

6. Suppose that instead of 1 bacterium, 50 bacteria land in your mouth. Write an equation that describes the number of bacteria, b, in this colony after h hours.

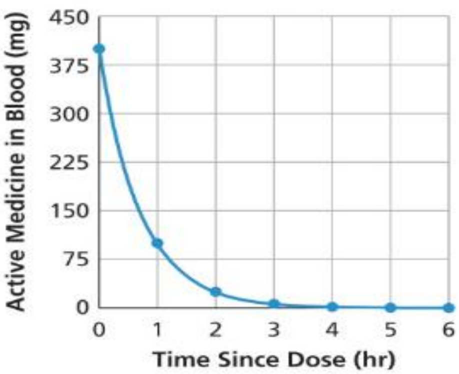
C. Loon Lake has a “killer plant” problem. Currently 5,000 square feet of the lake is covered with a plant that is killing all other aquatic life. The area covered is growing by a factor of 1.5 each year.

1. What is the equation for the growth of the plant?

2. The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered?

D. A dog receives a 400 milligram dose of flea medicine. The table and graph show the amount of medicine in the dog’s bloodstream each hour for 6 hours after the dose.

Breakdown of Medicine



Time Since Dose (hr)	Active Medicine in Blood (mg)
0	400
1	100
2	25
3	6.25
4	1.5625
5	0.3907
6	0.0977

1. How does the amount of active medicine in the dog’s blood change from one hour to the next?

2. Write an equation to model the decay of the flea medicine.

Name: ___________________________________ Date: ________________________ Hour: __Alg 1__

Unit 8 Day 6: Growth Factors vs. Growth rates

Focus Question: How are growth rates and growth factors related?

A. When Rodney first got his job in 1993, he earned $21,000 per year. At the end of each year, Rodney receives a 10% raise. *P* stands for Rodney's pay and *y* stands for years since 1993.

1. Using the equation $P(y) = 21000(10)^y$, what would his salary be this year?

2. Do you think the equation was correct? Explain.

3. If another way to write 10% is 0.1 how do you know the equation $P(y) = 21000(0.10)^y$, is also not correct?

B. Part A should have shown you that **percent increase** and **growth factor** are NOT the same thing. **When a <u>percent increase</u> is given, it is called a <u>growth rate</u>**. Growth **rates** are used quite frequently when speaking of financial investments. The table below gives Rodney's salary for the first 5 years after 1993.

Years since 1993	Salary in $
0	21000
1	23100
2	25410
3	27951
4	30746.10
5	33820.71

1. Find the **growth <u>factor</u>** (remember factor means what did you multiply by) for Rodney's salary. Explain how you got your answer.

2. Remember the growth rate was 10%. How are the growth rate and growth factor related?

A formula for exponential growth is $\boldsymbol{f(x) = A(1+r)^x}$ where *A* is the initial value,

r is the *<u>growth rate</u>* ,

x is the number of time intervals that have passed,

(1 + r) represents the *<u>growth factor</u>* or base.

C. Growth rates and factors

1. Find the growth factor associated with each growth rate.

   a. 75% b. 15% c. 30%

   d. 100% e. 150% f. 0%

2. Find the growth rate associated with each growth factor.

   a. 1.5 b. 1.25 c. 1.1 d. 1

D. When Sam was in seventh grade, his aunt gave him a stamp worth $2500. Sam considered selling the stamp, but his aunt told him that if he saved it, it would increase in value. Sam saved the stamp and its value increased by 6% each year for several years in a row.

1. Write an equation for the value of Sam's stamp for any year.

2. Make a table showing the value of the stamp each year for the five years after Sam's aunt gave it to him.

Year	Value
0	2500
1	
2	
3	
4	
5	

3. How many years will it take to double the value?

E. Mrs. Ramos started a college fund for her grandson. She used this calculation to predict the value of his fund several years from now:

$$\text{Value} = \$2000 \cdot 1.05 \cdot 1.05 \cdot 1.05 \cdot 1.05$$

1. What initial value, growth rate, growth factor, and number of years is Mrs. Ramos assuming?

2. If the value continues to increase at this rate, how much would the fund be worth in one more year?

Name: ___________________________________ Date: _________________________ Hour: __Alg 1__

Unit 8 Day 7: Decay rates

Focus Question: How are decay rates different from decay factors?

A. Not all exponential functions grow. The opposite of exponential growth is **exponential decay**. The function will decay if the dependent values constantly decrease by the same factor over a period of time. The factor is now called a **decay factor**.

1. What was the equation for exponential growth when a *growth factor* was given?

2. What was the equation for exponential growth when *growth rate* was given?

3. There is such a thing as a rate of decay. It is the percent at which something looses value over time. How do you think the equation will be different from the equation for exponential growth rates?

4. The equation for rate of decay is __________________________.

B. Decay Factors and Decay rates

1. Find the decay factor associated with each rate of decay.

a. 40% b. 35% c. 90%

2. Find the rate of decay associated with each decay factor

a. 0.4 b. 0.85 c. ¼

C. Writing equations of exponential growth and decay

1. A flea medicine breaks down at a rate of 20% per hour. This means that as each hour passes, 20% of the active medicine is used. This is the **rate of decay** of the medicine. The initial dose is 60 milligrams.
   a. What is the equation for how much medicine is in the bloodstream after so many hours?

   b. How much medication will be in the bloodstream after 2 hours?

   c. When will the medication be down to 10 milligrams?

2. New cars start to depreciate as soon as you drive them off the lot. Assuming you are going to own your car for more than 1 year, cars depreciate (lose value) at a rate of around 18% per year. If you received a Honda accord for your 15th birthday and were going to trade it in when you leave for college at the age of 18, how much can you expect as a trade in value?

3. A tree farm has begun to harvest a section of trees that was planted a number of years ago.

Supply of Trees

Year	0	1	2	3	4	5	6	7	8
Trees Remaining	10,000	9,502	9,026	8,574	8,145	7,737	7,350	6,892	6,543

a. Suppose the relationship between the year and the trees remaining is exponential. Approximate the decay factor for this relationship.

b. At what rate is the tree farm harvesting trees?

c. Write an equation for the relationship between time and trees remaining.

d. When will the number of trees be less than 100?

4. A half life is the amount of time that it takes for half of a substance to decay. DDT is a pesticide that was widely used until it was banned in 1972 because of its harm to the environment. DDT has a half life of 15 years.
   a. In what year will half of the DDT sprayed in 1972 be gone?

   b. How many half lives have passed since 1972?

   c. If a forest was sprayed with a substance that contained 100grams of DDT, just before it was banned, how much DDT is still in the forest?

Name: ___________________________________ Date: _________________________ Hour: ____Alg 1____

Unit 5 Day 8: Review

Focus Question: Do I remember everything about exponential functions?

1. Which table below shows exponential decay? ______ The function for it is _________________________

A.

x	y
0	500
1	525
2	550
3	575
4	600

B.

x	y
0	500
1	100
2	20
3	4
4	0.8

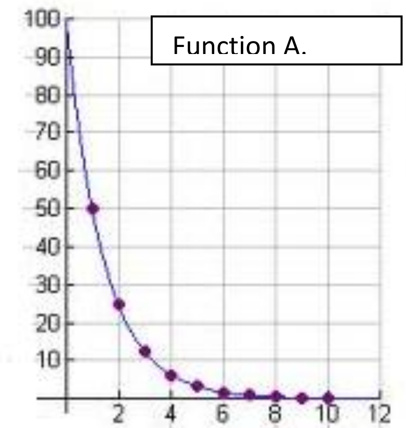
C.

x	y
0	500
1	400
2	300
3	200
4	100

D.

x	y
0	500
1	1,000
2	2,000
3	4,000
4	8,000

2. Two exponential decay functions are represented below. Write an equation for each. Then tell which has the greater **rate** of decay and explain.



Function B. A citrus orchard has 180 orange trees. A fungus attacks the trees. One month after the attack there were only 60 trees. Two months after the attack there were only 20 trees left.

3. Raquel decides to invest some money in the stock of a company. The company's prospectus says the stock averages a yearly rate of return of 4.5%. If Raquel invests $2500, how much will she have at the end of 6 years?